ON THE DEGREE OF AMBIGUITY OF FINITE AUTOMATA

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Abstract

We show that the degree of ambiguity of a nondeterministic finite automaton (NFA) with n states, if finite, is not greater than $2^{n \cdot \log_2 n + c_1 \cdot n}$ ($c_1 \cong 2.0566$). We present an algorithm which decides in polynomial time whether the degree of ambiguity of a NFA is finite or not. Additionally, the authors obtain in [14] a corresponding upper bound for the finite valuedness of a normalized finite transducer (NFT), and also a polynomial-time algorithm which decides whether the valuedness of a NFT is finite or not.

Introduction

Let x be an input string of a nondeterministic finite automaton (NFA) M, and also of a normalized finite transducer (NFT) M'. The degree of ambiguity of x in M is defined by the number of accepting paths for x in M. The valuedness of x in M' is defined by the number of different output strings on the accepting paths for x in M'. The degree of ambiguity of M (the valuedness of M') is the maximal degree of ambiguity (valuedness) of an input string of M (M') or is infinite, depending on whether or not a maximum exists.

The degree of ambiguity and the valuedness are structural parameters, which only recently received attention in connection with the equivalence problem for NFA's and NFT's: It is well-known that the equivalence problem for NFA's is PSPACE-complete [13], and that the equivalence of generalized sequential machines (NFT's without ε-(input) moves) is undecidable [5]. For any fixed integer d, however, the equivalence of NFA's with degree of ambiguity not greater than d can be tested in polynomial time [12]. From a generalization of the Ehrenfeucht conjecture follows that the equivalence of NFT's with finite valuedness is decidable [4].

Given any fixed integer d, it can be tested in polynomial time whether or not the degree of ambiguity of a NFA [12] (the valuedness of a NFT

[6]) is greater than d. Chan and Ibarra [2] exhibited two polynomial-space algorithms which decide whether the degree of ambiguity of a NFA is finite, and also whether it is greater than an arbitrary given integer. Moreover, they were able to prove that the latter problem is PSPACE-complete. Using a simple reduction, they obtained two exponential-space algorithms which solve both problems, if the degree of ambiguity is replaced by the valuedness of a sequential machine (a generalized sequential machine with length-preserving input-output relation).

In section 2 of this paper we show that the degree of ambiguity of a NFA with n states, if finite, is not greater than $2^{n\cdot\log_2 n + c_1\cdot n}$ ($c_1 \cong$ 2.0566). Such an explicit upper bound cannot be achieved with the methods used in [2]. In section 3 we introduce a criterion for the infinite degree of ambiguity of NFA's, which is testable in polynomial time (Independently of us, in [10] Ibarra and Ravikumar exhibit an equivalent criterion, which can be tested in exponential time). By simple reductions, in [14] the authors carry over both results to the degree of ambiguity of NFA's with ε-moves, and to the size of products of matrices over $\mathbb{N}_0$ (cf. [2]).

The ideas used are different from those in [2] and [10]. First of all, we show that it is sufficient to consider NFA's of a restricted type. Then, for every input string x, we investigate a graph which describes all accepting paths of x in the NFA, and we use "pumping arguments" in these graphs.

In [14] the authors extend the above-mentioned methods in order to achieve an upper bound for the finite valuedness of a NFT, and to define a criterion for the infinite valuedness of NFT's, which can be tested in polynomial time. The results are presented in section 4.

In the first section we summarize some definitions and notations.

1. Definitions

A nondeterministic finite automaton (NFA) is a 5-tuple $M=(Q,\Sigma,\delta,Q_0,F)$, where Q and Σ denote nonempty, finite sets of states and input symbols, $Q_0,F\subset Q$ denote sets of initial and final states, and δ is a subset of $Q\times\Sigma\times Q$.

A path of length m for x from p to q in M is a string $\pi = q_0x_1q_1\cdots x_mq_m \in Q(\Sigma Q)^*$ so that $x = x_1\ldots x_m \in \Sigma^*$, $p = q_0 \in Q$, $q = q_m \in Q$, and for all $i = 1,\dots,m$ $(q_{i-1},x_i,q_i) \in \delta$. We define $\hat{\delta} := \{(p,x,q) \in Q \times \Sigma^* \times Q \mid$ a path for x from p to q in M exists$\}$. Note that δ is a subset of $\hat{\delta}$. We rename $\hat{\delta}$ by δ.

A path π from p to q is called accepting path, if p is an initial state and q is a final state. $L(M)$, the language accepted by M, is the set of all $x \in \Sigma^*$, for which an accepting path in M exists.

The degree of ambiguity of $x \in \Sigma^*$ in M (short form: $da_M(x)$) is the number of all accepting paths for x in M. The degree of ambiguity of M (short form: $da(M)$) is the supremum of the degrees of ambiguity of all $x \in \Sigma^*$ in M, i. e. $da(M) = \sup\{da_M(x) \mid x \in \Sigma^*\}$.

The state $q \in Q$ is called useless, if it is on no accepting path in M. Useless states are irrelevant to the degree of ambiguity in M. If no state of M is useless, then M is called reduced.

A state $p \in Q$ is said to be connected with a state $q \in Q$ (short form: $p \longleftrightarrow q$), if paths from p to q and from q to p in M exist. Note that "$\longleftrightarrow$" is an equivalence relation on Q.

Let Q be divided into the equivalence classes $Q_1,\dots,Q_k$ w.r.t. "$\longleftrightarrow$". M is said to be of type 1, if the following holds:

$$Q_0 \subset Q_1 \;\&\; F \subset Q_k \;\&\; \delta \cap (Q\times\Sigma\times Q) \subset \bigcup_{i=1}^{k} Q_i\times\Sigma\times Q_i \cup \bigcup_{i=1}^{k-1} Q_i\times\Sigma\times Q_{i+1}.$$

M is said to be of type 2, if states $p_i,q_i \in Q_i$ $(i=1,\dots,k)$ exist such that the following holds:

$$Q_0 \subset \{p_1\} \;\&\; F \subset \{q_k\} \;\&\; \delta \cap (Q\times\Sigma\times Q) \subset \bigcup_{i=1}^{k} Q_i\times\Sigma\times Q_i \cup \bigcup_{i=1}^{k-1} \{q_i\}\times\Sigma\times\{p_{i+1}\}.$$

Let M be of type 2 such that $L(M) \neq \phi$, and let $x = x_1\dots x_m \in L(M)$. The graph $G_M(x) = (V,E)$ of the accepting paths for x in M is defined by

$$V := \{(q,i) \in Q\times\{0,\dots,m\} \mid (p_1,x_1\dots x_i,q) \in \delta \;\&\; (q,x_{i+1}\dots x_m,q_k) \in \delta\} \text{ and}$$

$$E := \{((p,i-1),(q,i)) \in V^2 \mid i \in \{1,\dots,m\} \;\&\; (p,x_i,q) \in \delta\}.$$

Note that the number of paths from $(p_1,0)$ to (q_k,m) in $G_M(x)$ equals the degree of ambiguity of x in M.

Let $M=(Q,\Sigma,\delta,Q_0,F)$ and $M'=(Q',\Sigma',\delta',Q_0',F')$ be two NFA's. M' is included in M (short form: $M' \subset M$), if $Q' \subset Q$, $\Sigma' \subset \Sigma$, $\delta' \subset \delta$, $Q_0' \subset Q_0$ and $F' \subset F$.

2. An Upper Bound for the Finite Degree of Ambiguity

In this section we construct for any NFA M with n states NFA's $M_1,\dots,M_N \subset M$ of type 2 such that $da(M) \leq \sum_{i=1}^{N} da(M_i)$ and $N \leq 2^n \cdot 3^{2n/3}$.
If M is of type 2 with $da(M) < \infty$, then we show: $da(M) \leq 2^{n\cdot\log_2 n}$.
This leads to the following result:

Theorem 1: Let M be a NFA with n states and finite degree of ambiguity. Then, the degree of ambiguity of M is not greater than $2^{n\cdot\log_2 n + c_1\cdot n}$, where $c_1 := 1 + (2/3)\cdot\log_2 3 \cong 2.0566$.

We do not know whether the upper bound of theorem 1 for the maximal finite degree of ambiguity of NFA's with n states is optimal. The best lower bound we know of is $2^{n\cdot(1+1/64)}$ (for details see [14]).

In the first lemma we consider the case of NFA's with one equivalence class w.r.t. "<--->".

Lemma 1: Let $M=(Q,\Sigma,\delta,Q_0,F)$ be a reduced NFA such that $da(M) < \infty$. Let $p \in Q$ be connected with $q \in Q$. Then, for any $x \in \Sigma^*$, there is at most one path for x from p to q in M.

Proof: Let $q_0 \in Q_0$, $q_F \in F$, and $u,v,w \in \Sigma^*$ such that (q_0,u,p), (q,w,q_F), $(q,v,p) \in \delta$. We assume that there are two different paths for x from p to q. This implies for any $i \geq 0$: $da_M(ux(vx)^i w) \geq 2^{i+1}$. Thus, $da(M) = \infty$. ↯ ∎

In the following we consider a NFA $M=(Q,\Sigma,\delta,Q_0,F)$. Proofs of the next two lemmas can be found in [14].

Lemma 2: Let M be a NFA with n states. We can construct NFA's $M_1,\dots,M_N \subset M$ of type 1 such that $N \leq 2^n$ and the following assertions are true:

(1) $da(M)<\infty \Rightarrow da(M) \leq \sum_{i=1}^{N} da(M_i) < \infty$.

(2) $da(M)=\infty \Rightarrow \exists\, i \in \{1,\dots,N\}: da(M_i) = \infty$.

Lemma 3: Let M be a NFA of type 1 with n states. We can construct NFA's $M_1,\dots,M_N \subset M$ of type 2 such that $N \leq 3^{2n/3}$ and the following assertions are true:

(1) $da(M)<\infty \Rightarrow da(M) \leq \sum_{i=1}^{N} da(M_i) < \infty$.

(2) $da(M)=\infty \Rightarrow \exists\, i \in \{1,\dots,N\}: da(M_i) = \infty$.

Lemma 4: Let M be a NFA of type 2 with n states such that $0 < da(M) < \infty$.

Then, $da(M) \leq 2^{n \cdot \log_2 n}$.

Proof: Let the equivalence classes $Q_1,\ldots,Q_k$ w.r.t. "$<--->$" on Q be numbered and the states $p_i, q_i \in Q_i$ (for $i=1,\ldots,k$) be given in correspondence with the definition of type 2. Since $da(M) > 0$, we know: $L(M) \neq \phi$, $Q_0=\{p_1\}$, $F=\{q_k\}$, M is reduced. We show by induction on k:

$$(*) \quad da(M) \leq 2^{n \cdot \lceil \log_2 k \rceil - k+1}$$

The lemma follows from (*) (for further details see [14]).

Proof of (*):

Base of induction: $k=1$. p_1 is connected with q_1. According to lemma 1 follows: $da(M) = 1$.

Induction step: Let $k \geq 2$. Define $l := \lceil k/2 \rceil$. Divide M into NFA's $M_1 = (\bigcup_{i=1}^{l} Q_i, \Sigma, \delta_1, \{p_1\}, \{q_l\})$ and $M_2 = (\bigcup_{i=l+1}^{k} Q_i, \Sigma, \delta_2, \{p_{l+1}\}, \{q_k\})$, where $\delta_1 := \delta \cap (\bigcup_{i=1}^{l} Q_i \times \Sigma \times Q_i \cup \bigcup_{i=1}^{l-1} \{q_i\} \times \Sigma \times \{p_{i+1}\})$ and

$\delta_2 := \delta \cap (\bigcup_{i=l+1}^{k} Q_i \times \Sigma \times Q_i \cup \bigcup_{i=l+1}^{k-1} \{q_i\} \times \Sigma \times \{p_{i+1}\})$.

M_1, M_2 are NFA's of type 2 with $n_1 := \sum_{i=1}^{l} \#Q_i$ and $n_2 := \sum_{i=l+1}^{k} \#Q_i$ states such that $0 < da(M_1) < \infty$ and $0 < da(M_2) < \infty$.

Let $x=x_1\ldots x_m \in L(M)$. Consider in the graph $G_M(x)=(V,E)$ the set D of all edges "from Q_l to Q_{l+1}", i.e. $D = \{((q_l,j-1),(p_{l+1},j)) \in E \mid j \in \{1,\ldots,m\}\}$. Define $J := \{j \in \{1,\ldots,m\} \mid ((q_l,j-1),(p_{l+1},j)) \in E\}$. We observe:

$$da_M(x) = \sum_{j \in J} da_{M_1}(x_1\ldots x_{j-1}) \cdot da_{M_2}(x_{j+1}\ldots x_m).$$

From the induction hypothesis follows:

$$da_M(x) \leq \sum_{j \in J} 2^{n_1 \cdot \lceil \log_2 \lceil k/2 \rceil \rceil - \lceil k/2 \rceil + 1} \cdot 2^{n_2 \cdot \lceil \log_2 \lfloor k/2 \rfloor \rceil - \lfloor k/2 \rfloor + 1}$$

$$\leq \#J \cdot 2^{n \cdot \lceil \log_2 (k/2) \rceil - k+2} = \#J \cdot 2^{n \cdot \lceil \log_2 k \rceil - n-k+2}.$$

Note that $\lceil \log_2 \lceil k/2 \rceil \rceil = \lceil \log_2 (k/2) \rceil$. Therefore, it is sufficient to show: $\#D = \#J \leq 2^{n-1}$.

We assume that $\#J > 2^{n-1}$. Consider for all $j \in J$ $A_j := \{q \in Q \mid (q,j) \in V\}$. Since $\#J > 2^{n-1}$ and, for all $j \in J$, $p_{l+1} \in A_j \subset Q$, $j_1, j_2 \in J$ exist such that $j_1 < j_2$ and $A_{j_1} = A_{j_2} =: A$.

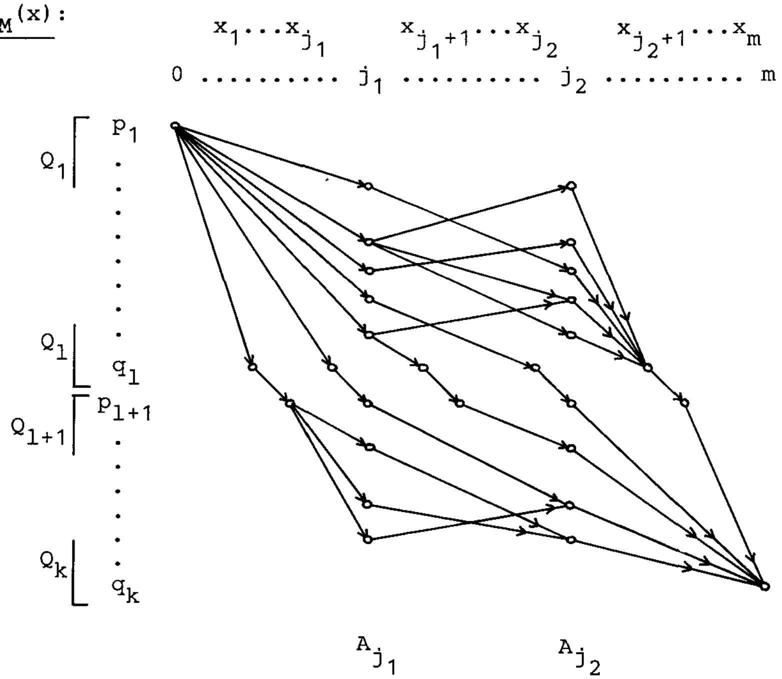
Let $t \in \mathbb{N}$. Define $y_t := x_1\ldots x_{j_1} (x_{j_1+1}\ldots x_{j_2})^t x_{j_2+1}\ldots x_m$. For all $\tau \in \{0,\ldots,t\}$ and all $q \in A$ we observe:

$$(p_1 , x_1 \dots x_{j_1} (x_{j_1+1} \dots x_{j_2})^{\tau} , q) \in \delta \ \&$$

$$(q , (x_{j_1+1} \dots x_{j_2})^{\tau} x_{j_2+1} \dots x_m , q_k) \in \delta.$$

Moreover, there is a path for $x_{j_1+1} \dots x_{j_2}$ from some state in $A \cap \bigcup_{i=1}^{l} Q_i$ to p_{l+1} in M (via $(q_l, x_{j_2}, p_{l+1}) \in \delta$). From this follows: $da_M(y_t) \geqq t$. Thus, $da(M) = \infty$. ↯

$\underline{G_M(x)}$:



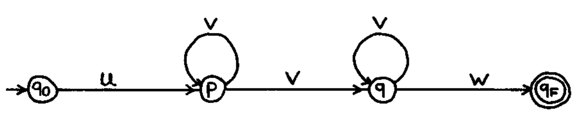
∎

This completes the proof of theorem 1.

3. A Criterion for the Infinite Degree of Ambiguity, Which is Decidable in Polynomial Time

Let $M=(Q,\Sigma,\delta,Q_0,F)$ be a NFA. Consider the following criterion for M:

$$\text{(DA)} \quad \begin{cases} \exists\, q_0 \in Q_0 \quad \exists\, p,q, \in Q \quad \exists\, q_F \in F \quad \exists\, u,v,w \in \Sigma^* \ : \quad (q_0,u,p) \in \delta \\ \& \quad (p,v,p),(p,v,q),(q,v,q) \in \delta \quad \& \quad (q,w,q_F) \in \delta \quad \& \quad p \neq q. \end{cases}$$



If M complies with (DA), then we observe that for all $i \in \mathbb{N}$ $da_M(uv^iw) \geq i$, i.e. $da(M) = \infty$. If da(M) is infinite, then, according to lemma 2 and 3, there is a NFA $M' \subset M$ of type 2 such that da(M') is infinite. In this section we show that M' complies with (DA), and hence M complies with (DA), too. This leads to the following result:

Theorem 2: Let M be a NFA. The degree of ambiguity of M is infinite, if and only if M complies with (DA).

We can decide in polynomial time whether or not a NFA complies with (DA): Let $M=(Q,\Sigma,\delta,Q_0,F)$ be a reduced NFA with n states. Consider the directed graph $G_3 = (Q^3,E_3)$, where $E_3 :=$
$= \{ ((p_1,p_2,p_3),(q_1,q_2,q_3)) \in Q^3 \times Q^3 \mid \exists\ a \in \Sigma \ \ \forall\ i \in \{1,2,3\} :$
$(p_i,a,q_i) \in \delta \}$. G_3 allows us to rewrite (DA) as follows:

$\exists\ p,q \in Q :\ \ p \neq q\ \ \&\ \$ in G_3 a path from (p,p,q) to (p,q,q) exists.

Thus, with well-known graph algorithms we can test in time $O(n^6 \cdot \#\Sigma + n^8)$ (on a RAM using the uniform cost criterion) whether or not M complies with (DA) (see [1]). With theorem 2 follows:

Theorem 3: Let M be a NFA with n states and an input alphabet Σ. It is decidable in time $O(n^6 \cdot \#\Sigma + n^8)$ whether or not the degree of ambiguity of M is infinite.

We remark that the time bound in theorem 3 can be improved to $O(n^6 \cdot \#\Sigma)$ (see [14]). With theorem 2 of [2] and with theorem 1 the exact (finite) degree of ambiguity of a NFA can be computed in polynomial space.

The following lemma completes the proof of theorem 2.

Lemma 5: Let M be a NFA of type 2 such that da(M) is infinite. Then, M complies with (DA).

Proof: Let $M=(Q,\Sigma,\delta,Q_0,F)$. Let the equivalence classes $Q_1,\ldots,Q_k$ w.r.t. "<--->" on Q be numbered and the states $p_i,q_i \in Q_i$ (for $i=1,\ldots,k$) be given in correspondence with the definition of type 2. Since $da(M) = \infty$, we know: $Q_0=\{p_1\}$, $F=\{q_k\}$, M is reduced.

Case 1: There are $p',q' \in Q$ and $y \in \Sigma^*$ such that p' is connected with q', and two different paths for y from p' to q' in M exist.

This implies: $\exists\ p,q \in Q\ \ \exists\ y_1,y_2,y_3 \in \Sigma^* :\ \ p \neq q\ \ \&\ \ y = y_1y_2\ \ \&$
$(p',y_1,p),(p',y_1,q),(p,y_2,q'),(q,y_2,q') \in \delta\ \ \&\ \ (q',y_3,p') \in \delta$. Define

$v := y_2y_3y_1$, then: $\{p,q\}\times\{v\}\times\{p,q\}\subset\delta$. Since M is reduced, M complies with (DA). .

Case 2: For all $i\in\{1,\dots,k\}$, all $p',q'\in Q_i$, and all $y\in\Sigma^*$ there is at most one path for y from p' to q' in M.

Let $x=x_1\dots x_m\in L(M)$ and $l\in\{1,\dots,k-1\}$ (note: $k\geq 2$). Consider in the graph $G_M(x)=(V,E)$ the set $D_l(x)$ of all edges "from Q_l to Q_{l+1}", i.e. $D_l(x) = \{((q_l,j-1),(p_{l+1},j))\in E \mid j\in\{1,\dots,m\}\}$. Define $n := \#Q$. We are able to choose $x\in L(M)$ and $l\in\{1,\dots,k-1\}$ so that $\#D_l(x) > 2^{n-1}$. Otherwise we would have, because of case 2:

$$\forall\ x\in L(M) : da_M(x) \leq \prod_{l=1}^{k-1} \#D_l(x) \leq 2^{(n-1)\cdot(k-1)},$$

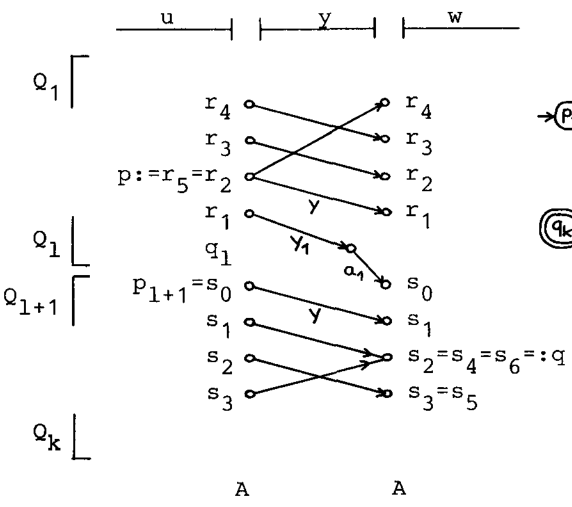
i.e. $da(M) < \infty$. ↯

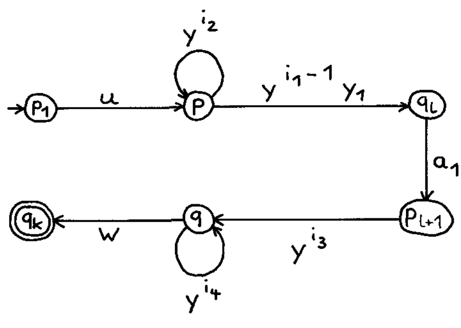
Just like in the proof of lemma 4 we can find a decomposition $x = uyw$ with the following properties:

$u,y\neq\varepsilon$, $((q_l,|u|-1),(p_{l+1},|u|))\in E$, $((q_l,|uy|-1),(p_{l+1},|uy|))\in E$, $A := \{q\in Q \mid (q,|u|)\in V\} = \{q\in Q \mid (q,|uy|)\in V\}$.

We construct states $r_i\in A$ ($i\geq 1$) as follows: Let $a_1\in\Sigma$ so that $y = y_1a_1$. Choose $r_1\in A$ such that $(r_1,y_1,q_l)\in\delta$. Choose $r_i\in A$ such that $(r_i,y,r_{i-1})\in\delta$ $(i=2,3,\dots)$. There are $i_1,i_2\in\mathbb{N}$ such that $r_{i_1} = r_{i_1+i_2} =: p$.

We construct states $s_i\in A$ ($i\geq 0$) as follows: Define $s_0 := p_{l+1}\in A$. Choose $s_i\in A$ such that it holds: $(s_{i-1},y,s_i)\in\delta$ & $(\forall\ j\in\{1,\dots,i\}$: $s_{i-1}=s_{j-1} \Rightarrow s_i=s_j)$ $(i=1,2,\dots)$. There are $i_3\in\mathbb{N}_0$ and $i_4\in\mathbb{N}$ such





that $s_{i_3} = s_{i_3+i_4} =: q$ and $i_1+i_3=0 \bmod i_2$.
In conclusion, we know:

$$(p\ ,\ y^{i_2}\ ,\ p) \in \delta\ ,\quad (p\ ,\ y^{i_1-1}y_1\ ,\ q_l) \in \delta\ ,\ (q_l\ ,\ a_1\ ,\ p_{l+1}) \in \delta\ ,$$
$$(p_{l+1}\ ,\ y^{i_3}\ ,\ q) \in \delta\ ,\ (q\ ,\ y^{i_4}\ ,\ q) \in \delta.$$

p and q are different, because otherwise q_l would be connected with p_{l+1}. Choose $j_1 \geq (i_1+i_3)/i_2$ so that $j_1 = 0 \bmod i_4$, and define $v := y^{i_2 \cdot j_1}$. Then, we have:

$$(p,v,p) \in \delta\ ,\quad (p,v,q) \in \delta\ ,\quad (q,v,q) \in \delta.$$

Moreover, we know that $p_1 \in Q_0$, $(p_1,u,p) \in \delta$, $q_k \in F$ and $(q,w,q_k) \in \delta$. Hence, M complies with (DA). ■

We remark that the proof of lemma 2 contains the core of a brief proof of a criterion for the exponential degree of ambiguity, which was introduced in [10] (for details see [14]).

4. Results on Valuedness

We are able to modify the theorems 1 and 3 so that the degree of ambiguity of a NFA is replaced by the valuedness of a NFT. The outcome is stated in this section, the proofs are given in [14].

A normalized finite transducer (NFT) is a 6-tuple $M = (Q,\Sigma,\Delta,\delta,Q_0,F)$, where Q,Σ,Q_0 and F have the same meaning as in a NFA, Δ is a nonempty, finite output alphabet, and δ is a finite subset of $Q \times (\Sigma \cup \{\varepsilon\}) \times \Delta^* \times Q$. A path with input x and output z in M is a string $\pi = q_0 \prod_{i=1}^{m} x_i z_i q_i \in Q((\Sigma \cup \{\varepsilon\})\Delta^* Q)^*$ such that $x=x_1 \ldots x_m \in \Sigma^*$, $z=z_1 \ldots z_m \in \Delta^*$, and for all $i=1,\ldots,m$ $(q_{i-1},x_i,z_i,q_i) \in \delta$. Accepting paths are defined just like in a NFA.

The valuedness of $x \in \Sigma^*$ in M (short form: $val_M(x)$) is the number of all strings $z \in \Delta^*$ so that an accepting path with input x and output z exists in M. The valuedness of M (short form: val(M)) is the supremum of the set $\{val_M(x) \mid x \in \Sigma^*\}$. Moreover, we define

$$\begin{aligned}
val(\delta) &:= \max(\{1\} \cup \{\#\, \delta \cap (\{p\} \times \{a\} \times \Delta^* \times \{q\}) \mid p,q \in Q\ ,\ a \in \Sigma \cup \{\varepsilon\}\}),\\
diff(\delta) &:= \max(\{0\} \cup \{\ ||z_1|-|z_2||\ \mid \exists a \in \Sigma\ \ \forall i=1,2\ \ \exists p_i,q_i \in Q :\\
&\qquad (p_i,a,z_i,q_i) \in \delta\} \cup \{|z| \mid \exists p,q \in Q : (p,\varepsilon,z,q) \in \delta\}).
\end{aligned}$$

<u>Theorem 4</u>: Let $M = (Q,\Sigma,\Delta,\delta,Q_0,F)$ be a NFT with n states and finite

valuedness. Then, the valuedness of M is not greater than
$2^{(n-5)\cdot\log_2 n + c_3\cdot(n-1)} \cdot val(\delta)^{n-1} \cdot (1+diff(\delta))^{n-1} \cdot \#\Delta^{n^4\cdot diff(\delta)}$,
where $c_3 := 21 + (2/3)\cdot\log_2 3 \cong 22.0566$.

Theorem 5: Let M be a NFT. It is decidable in polynomial time whether or not the valuedness of M is infinite.

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