# Formal semantics

Let $fn(M)$ and $fn(P)$ be the set of free names in term $M$ and process $P$ respectively. Let $fv(M)$ and $fv(P)$ be the set of free variables in term $M$ and process $P$ respectively.

Closed processes are processes without any free variables.

## Reaction relation:

A process is like a chemical solution of molecules waiting to react.

$$\overline{m}\langle N\rangle.P \mid m(x).Q \rightarrow P \mid Q[N/x]$$

Reduction relation $>$ on closed processes:

$$
\begin{aligned}
!P &> P \mid !P \\
[M\ is\ M]P &> P \\
let\ (x,y) = (M,N)\ in\ P &> P[M/x][N/y] \\
case\ 0\ of\ 0 : P\ suc(x) : Q &> P \\
case\ \ suc(M)\ of\ 0 : P\ suc(x) : Q &> Q[M/x] \\
case\ \{M\}_N\ of\ \{x\}_N\ in\ P &> P[M/x]
\end{aligned}
$$

Structural equivalence on closed processes:

$$
\begin{aligned}
P \mid \mathbf{0} &\equiv P \\
P \mid Q &\equiv Q \mid P \\
P \mid (Q \mid R) &\equiv (P \mid Q) \mid R \\
(\nu m)(\nu n)P &\equiv (\nu n)(\nu m)P \\
(\nu n)\mathbf{0} &\equiv \mathbf{0} \\
(\nu n)(P \mid Q) &\equiv P \mid (\nu n)Q \text{ if } n \notin fn(P)
\end{aligned}
$$

$$\frac{P > Q}{P \equiv Q}$$

$$\frac{}{P \equiv P} \qquad \frac{P \equiv Q}{Q \equiv P} \qquad \frac{P \equiv Q \quad Q \equiv R}{P \equiv R}$$

$$\frac{P \equiv P'}{P \mid Q \equiv P' \mid Q} \qquad \frac{P \equiv P'}{(\nu m)P \equiv (\nu m)P'}$$

The complete reaction rules:

$$\overline{m}\langle N\rangle.P \mid m(x).Q \rightarrow P \mid Q[N/x]$$

$$\frac{P \equiv P' \quad P' \rightarrow Q' \quad Q' \equiv Q}{P \rightarrow Q}$$

$$\frac{P \rightarrow P'}{P \mid Q \rightarrow P' \mid Q} \qquad \frac{P \rightarrow P'}{(\nu n)P \rightarrow (\nu n)P'}$$

$$A \longrightarrow S : \{K_{AB}\}_{K_{AS}} \text{ on } c_{AS}$$

$$S \longrightarrow B : \{K_{AB}\}_{K_{SB}} \text{ on } c_{SB}$$

$$A \longrightarrow B : \{M\}_{K_{AB}} \text{ on } c_{AB}$$

$$\begin{array}{rcl}
A(M) & \triangleq & (\nu K_{AB})(\overline{c_{AS}}\langle\{K_{AB}\}_{K_{AS}}\rangle \\
 & & .\overline{c_{AB}}\langle\{M\}_{K_{AB}}\rangle.\mathbf{0} \\
S & \triangleq & c_{AS}(x).case\ x\ of\ \{y\}_{K_{AS}}\ in\ \overline{c_{SB}}\langle\{y\}_{K_{SB}}\rangle.\mathbf{0} \\
B & \triangleq & c_{SB}(x).case\ x\ of\ \{y\}_{K_{SB}}\ in \\
 & & c_{AB}(z).case\ z\ of\ \{w\}_{y}\ in\ F(w) \\
Inst(M) & \triangleq & (\nu K_{AS})(\nu K_{SB})(A(M) \mid S \mid B)
\end{array}$$

$$
\begin{array}{rcl}
Inst(M) & \equiv & (\nu K_{AS})(\nu K_{SB})(A(M) \mid S \mid B) \\
& \longrightarrow & (\nu K_{AS})(\nu K_{SB})(\nu K_{AB}) \\
& & (\overline{c_{AB}}\langle\{M\}_{K_{AB}}\rangle.\mathbf{0} \mid \overline{c_{SB}}\langle\{K_{AB}\}_{K_{SB}}\rangle.\mathbf{0} \mid B) \\
& \longrightarrow & (\nu K_{AS})(\nu K_{SB})(\nu K_{AB}) \\
& & (\overline{c_{AB}}\langle\{M\}_{K_{AB}}\rangle.\mathbf{0} \mid \\
& & c_{AB}(z).case\ z\ of\ \{w\}_{K_{AB}}\ in\ F(w)) \\
& \longrightarrow & (\nu K_{AS})(\nu K_{SB})(\nu K_{AB})F(M)
\end{array}
$$

## Testing equivalence

For this equivalence we are interested in the channels on which a process may communicate.

A barb is an element $\beta \in \{m, \overline{m}\}$ where $m$ is a name.

We write $P \downarrow \beta$ to say that the closed process $P$ can input or output immediately on the barb $\beta$. We say that $P$ exhibits barb $\beta$.

$$m(x).P \downarrow m \qquad \overline{m}\langle M \rangle.P \downarrow \overline{m}$$

$$\frac{P \downarrow \beta}{P \mid Q \downarrow \beta} \qquad \frac{P \downarrow \beta \quad \beta \notin \{m, \overline{m}\}}{(\nu m)P \downarrow \beta} \qquad \frac{P \equiv Q \quad Q \downarrow \beta}{P \downarrow \beta}$$

We write $P \Downarrow \beta$ to say that $P$ exhibits $\beta$ after some reactions.

$$\frac{P \downarrow \beta}{P \Downarrow \beta} \qquad \frac{P \rightarrow Q \quad Q \Downarrow \beta}{P \Downarrow \beta}$$

a test is a closed process $R$ and a barb $\beta$. A closed process $P$ passes the test iff $(P \mid R) \Downarrow \beta$.

The testing equivalence is defined as:

$P \simeq Q \triangleq$ for any test $(R, \beta)$, $(P \mid R) \Downarrow \beta$ iff $(Q \mid R) \Downarrow \beta$.

$$A \longrightarrow B : M \qquad \text{on } c_{AB}$$

$$\begin{aligned} A(M) \;&\triangleq\; \overline{c_{AB}}\langle M\rangle.\mathbf{0} \\ B \;&\triangleq\; c_{AB}(x).\mathbf{0} \\ Inst(M) \;&\triangleq\; (\nu c_{AB})(A(M) \mid B) \end{aligned}$$

Secrecy property: $Inst(M) \simeq Inst(M')$ for all $M, M'$.

i.e., for any process $R$ and barb $\beta$,

$(Inst(M) \mid R) \Downarrow \beta$ iff $(Inst(M') \mid R) \Downarrow \beta$.

Actually the only barbs exhibited are those by the process $R$.

But if $A$ and $B$ communicate on unrestricted channels:

$$
\begin{aligned}
A(M) &\triangleq \overline{c_{AB}}\langle M\rangle.\mathbf{0} \\
B &\triangleq c_{AB}(x).\mathbf{0} \\
Inst(M) &\triangleq A(M) \mid B
\end{aligned}
$$

Let $m$ be some message supposed to be secret. Then consider the process

$$R \triangleq c_{AB}(x).[x\ is\ m]\overline{d}\langle x\rangle.\mathbf{0}$$

We have $(Inst(m) \mid R) \Downarrow d$
but not $(Inst(M) \mid R) \Downarrow d$ for $m \neq M$.

$$
\begin{aligned}
A(M) &\triangleq \overline{c_{AB}}\langle M\rangle.\mathbf{0} \\
B &\triangleq c_{AB}(x).F(x) \\
Inst(M) &\triangleq (\nu c_{AB})(A(M) \mid B)
\end{aligned}
$$

For any process $R$, the barbs exhibited by the process $Inst(M) \mid R$ are exactly those exhibited by the process $F(M) \mid R$, hence we have the required secrecy property:

If $F(M) \simeq F(M')$ for all $M, M'$
then $Inst(M) \simeq Inst(M')$ for all $M, M'$

Authenticity:

$$\begin{aligned} A(M) &\triangleq \overline{c_{AB}}\langle M \rangle.\mathbf{0} \\ B_{spec}(M) &\triangleq c_{AB}(x).F(M) \\ Inst_{spec}(M) &\triangleq (\nu c_{AB})(A(M) \mid B_{spec}(M)) \end{aligned}$$

The barbs exhibited by the process $Inst_{spec}(M) \mid R$ are exactly those exhibited by the process $F(M) \mid R$, hence we have the required authenticity property:

$$Inst(M) \simeq Inst_{spec}(M) \text{ for all } M$$

In case of encryption:

$$P(M) \triangleq (\nu K)\overline{c}\langle\{M\}_K\rangle.\mathbf{0}$$

Secrecy is again preserved: $P(M) \simeq P(M')$ for all $M, M'$.

This is because the key $K$ is restricted. No other process $R$ may decrypt using this key. Hence whatever actions he may take using the message $\{M\}_K$ he may also take similar actions using the message $\{M'\}_K$.

# Some approximation techniques for protocol analysis

The protocol security problem is undecidable.

Hence we do approximate analysis of protocols.

- Unsafe approximation: detect a few attacks, but not necessarily all. Useful while developing protocols.
  - bounding the number of sessions
  - bounding the size of messages
  - . . .
- Safe approximation: detect all attacks. Sometimes false attacks may be detected. Useful for certifying protocols.

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  - ...
- Safe approximation: detect all attacks. Sometimes false attacks may be detected. Useful for certifying protocols.
  For the secrecy problem: over-approximate the intruder's knowledge

A common approximation is to let nonces be non-fresh in our modeling of the protocol.

Insecure protocol remains insecure after this abstraction.

Proof idea: take an attack in the multiset rewriting notation.
Show that systematically replacing a nonce by some other term produces a valid attack (except for the freshness condition on nonces).

Hence we choose a small number of nonces which are used repeatedly in several sessions.

Typical approximation: use only a finitely many nonces.

The public key Needham-Schroeder example:

$$\begin{array}{ll} 1. & A \longrightarrow B : \{A, N_a\}_{K_b} \\ 2. & B \longrightarrow A : \{N_a, N_b\}_{K_a} \\ 3. & A \longrightarrow B : \{N_b\}_{K_b} \end{array}$$

Following the results on reduction of number of agents, we first fix two honest agents $A, B$ and one dishonest agent $C$.

Choose a finite set of nonces $n^1_{xy}, n^2_{xy}, n^1_{yx}, n^2_{yx}$ for distinct agents $x$ and $y$.

Choose three keys $K_a, K_b, K_c$.

We have rules to define (an over-approximation of) the set of messages known to the intruder.

1. $A \longrightarrow B : \{A, N_a\}_{K_b}$
2. $B \longrightarrow A : \{N_a, N_b\}_{K_a}$
3. $A \longrightarrow B : \{N_b\}_{K_b}$

$$I(\{A, n_{ab}^1\}_{K_b})$$
$$I(\{A, n_{ac}^1\}_{K_c})$$
$$I(\{B, n_{ba}^1\}_{K_a})$$
$$I(\{B, n_{bc}^1\}_{K_c})$$
$$I(\{C, n_{ca}^1\}_{K_a})$$
$$I(\{C, n_{cb}^1\}_{K_b})$$

These can be written as push and pop rules.

The rule $I(\{A, n^1_{ab}\}_{K_b})$ is written as

$$\begin{aligned}
&\rightarrow q_1(A)\\
&\rightarrow q_2(n^1_{ab})\\
&\rightarrow q_3(K_b)\\
q_1(x), q_2(y) &\rightarrow q_4(\langle x, y\rangle)\\
q_4(x), q_3(y) &\rightarrow I(\{x\}_y)
\end{aligned}$$

for fresh states $q_1, q_2, q_3, q_4$.

The above are all push rules.

1. $A \longrightarrow B : \{A, N_a\}_{K_b}$
2. $B \longrightarrow A : \{N_a, N_b\}_{K_a}$
3. $A \longrightarrow B : \{N_b\}_{K_b}$

$$
\begin{aligned}
I(\{A, x\}_{K_b}) &\rightarrow I(\{x, n^2_{ab}\}_{K_a}) \\
I(\{A, x\}_{K_c}) &\rightarrow I(\{x, n^2_{ac}\}_{K_a}) \\
I(\{B, x\}_{K_a}) &\rightarrow I(\{x, n^2_{ba}\}_{K_b}) \\
I(\{B, x\}_{K_c}) &\rightarrow I(\{x, n^2_{bc}\}_{K_b}) \\
I(\{C, x\}_{K_a}) &\rightarrow I(\{x, n^2_{ca}\}_{K_c}) \\
I(\{C, x\}_{K_b}) &\rightarrow I(\{x, n^2_{cb}\}_{K_c})
\end{aligned}
$$

The rule $I(\{A, x\}_{K_b}) \longrightarrow I(\{x, n_{ab}^2\}_{K_a})$ can be written as:

$$
\begin{array}{rlr}
 & \longrightarrow p_1(K_b) & \text{(push)} \\
I(\{y\}_z), p_1(z) & \longrightarrow p_2(y) & \text{(pop)} \\
 & \longrightarrow p_3(A) & \text{(push)} \\
p_2(\langle y', x\rangle), p_3(y') & \longrightarrow p_4(x) & \text{(pop)} \\
 & \longrightarrow p_5(n_{ab}^2) & \text{(push)} \\
p_4(x), p_5(x') & \longrightarrow p_6(\langle x, x'\rangle) & \text{(pop)} \\
 & \longrightarrow p_7(K_a) & \text{(push)} \\
p_6(x''), p_7(x''') & \longrightarrow I(\{x''\}_{x'''}) & \text{(pop)}
\end{array}
$$

1. $A \longrightarrow B : \{A, N_a\}_{K_b}$

2. $B \longrightarrow A : \{N_a, N_b\}_{K_a}$

3. $A \longrightarrow B : \{N_b\}_{K_b}$

$$I(\{n^1_{ab}, x\}_{K_a}) \longrightarrow I(\{x\}_{K_b})$$

$$I(\{n^1_{ac}, x\}_{K_a}) \longrightarrow I(\{x\}_{K_c})$$

$$I(\{n^1_{ba}, x\}_{K_b}) \longrightarrow I(\{x\}_{K_c})$$

$$I(\{n^1_{bc}, x\}_{K_b}) \longrightarrow I(\{x\}_{K_a})$$

$$I(\{n^1_{ca}, x\}_{K_c}) \longrightarrow I(\{x\}_{K_a})$$

$$I(\{n^1_{cb}, x\}_{K_c}) \longrightarrow I(\{x\}_{K_b})$$

As usual we have the intruder's initial knowledge:

$I(n^1_{ca})$ $I(n^1_{cb})$ $I(n^2_{ac})$ $I(n^2_{bc})$ $I(K_a)$ $I(K_b)$

$I(K_c)$ $I(K_c^{-1})$ $I(A)$ $I(B)$ $I(C)$

The rules for intruder actions:

$$\begin{aligned} I(x), I(y) &\rightarrow I(\{x\}_y) \\ I(\{x\}_k), I(k^{-1}) &\rightarrow I(x) \\ &\ldots \end{aligned}$$

Example secrecy question : is nonce $n^1_{ab}$ accepted at state $I$?

Security of this abstract protocol implies security of the real protocol.

Finally all protocols need not be translatable to push and pop rules.

E.g. rules like $q_1(f(x, x)), q_2(x) \longrightarrow q(x)$ are not pop rules.

Indeed the secrecy question is undecidable even for protocols without nonces.

Less severe abstractions are also possible.

E.g. the nonces may be a function not just of user names, but also of previous messages exchanged.

$$
\begin{array}{ll}
1. & A \longrightarrow B : \{A, N_a\}_{K_b} \\
2. & B \longrightarrow A : \{N_a, N_b\}_{K_a} \\
3. & A \longrightarrow B : \{N_b\}_{K_b}
\end{array}
$$

$$I(\{A, x\}_{K_b}) \ \rightarrow I(\{x, n^2_{ab}(x)\}_{K_a})$$

$$\ldots$$

The second nonce $n^2_{ab}(x)$ depends on message $x$ received.

Nonces may be non-fresh, but infinitely many of them may be used.

# Extending the Dolev-Yao model with equations

Sometimes an accurate analysis of protocols requires considering special properties of underlying operations.

E.g. for the Diffie-Hellman key exchange we required special properties of modular exponentiation.

$$(g^x)^y = (g^y)^x$$

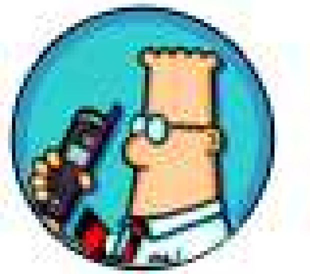
Other operations that are often used are e.g. XOR.

Sometimes there are attacks against protocols based on these special properties of the underlying operations.

## Group key agreement protocols

Generalization of the Diffie-Hellman key exchange to several participants.

A



B

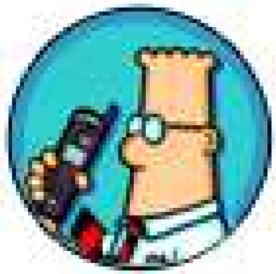
C

## Group key agreement protocols

Generalization of the Diffie-Hellman key exchange to several participants.

A

$$\xrightarrow{\alpha^{Na}}$$

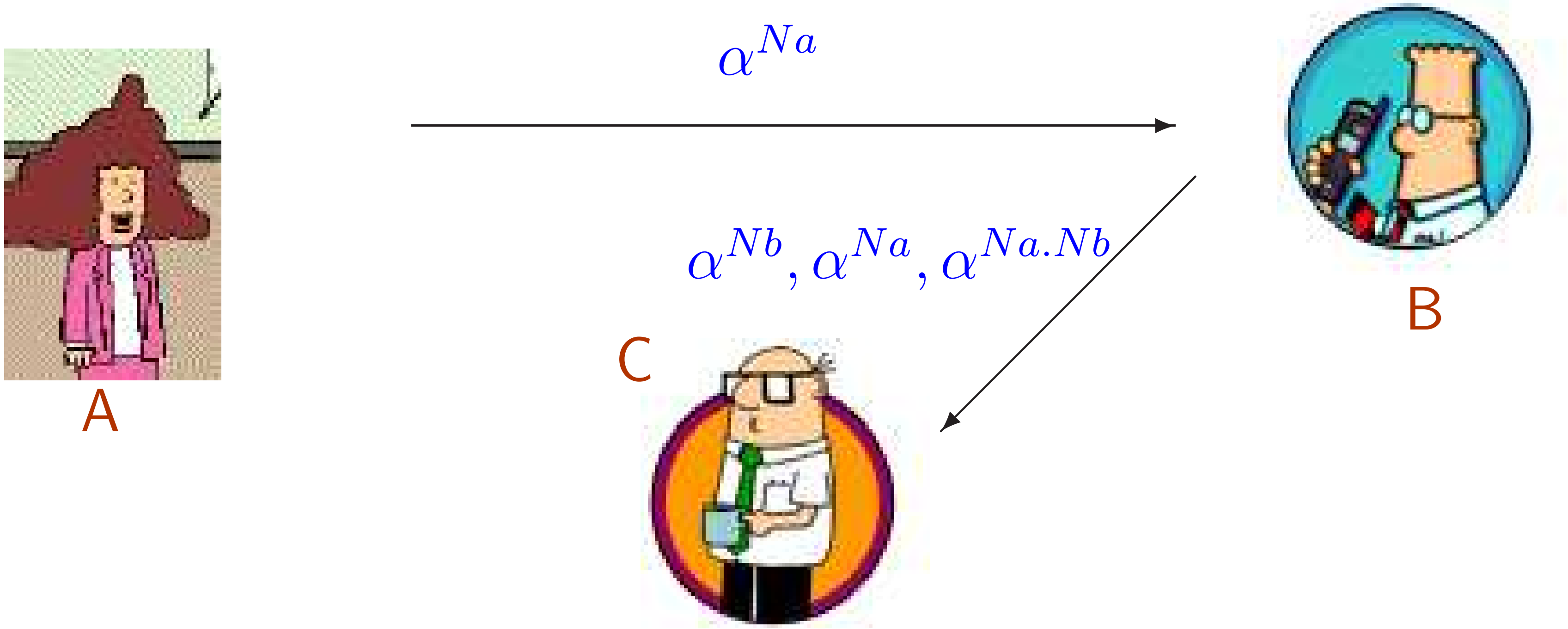


B

C

## Group key agreement protocols

Generalization of the Diffie-Hellman key exchange to several participants.



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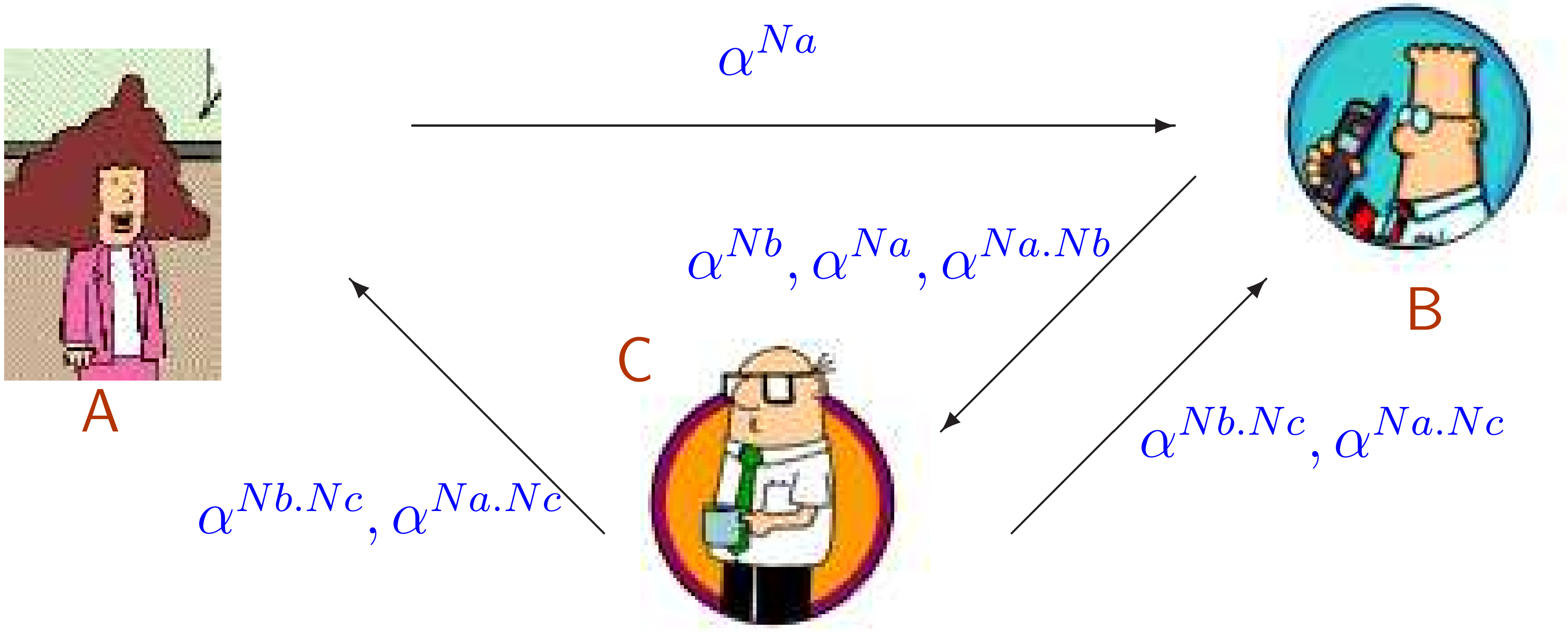

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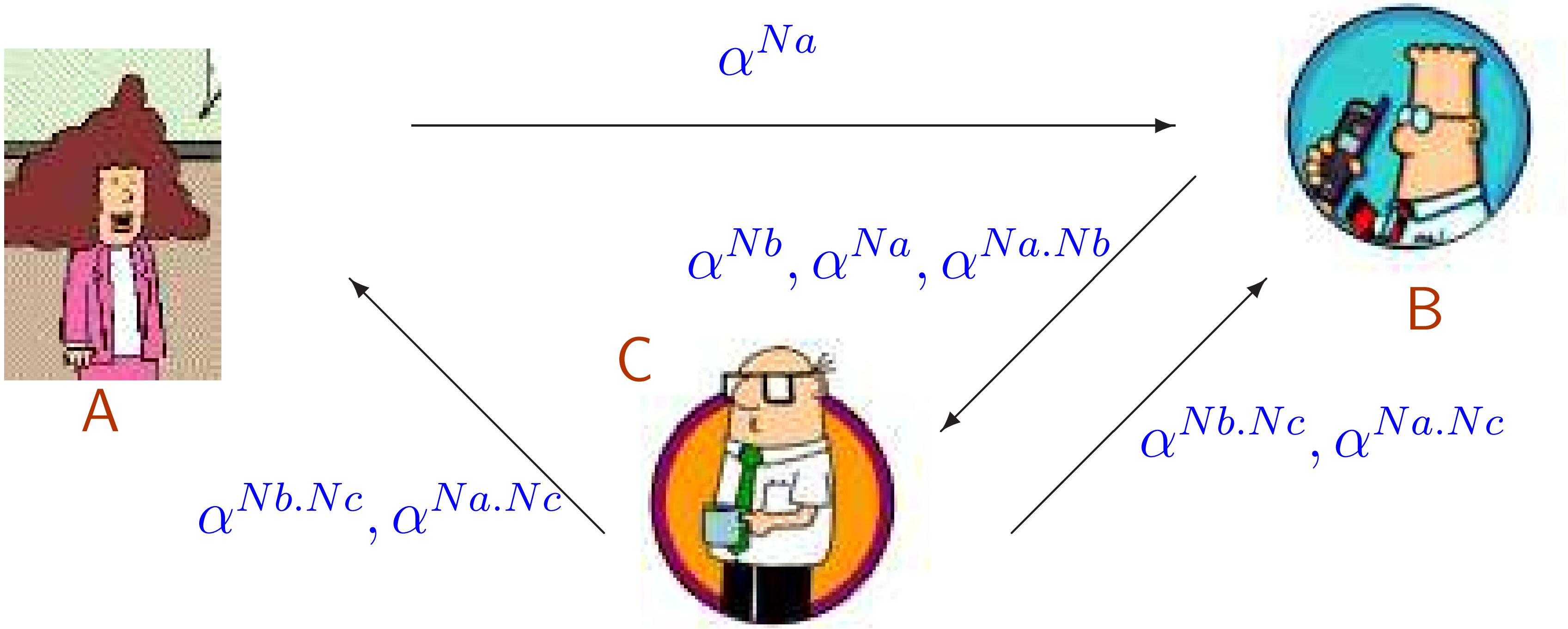


Group key $\alpha^{Na.Nb.Nc}$ is then computed by each participant

## Group key agreement protocols

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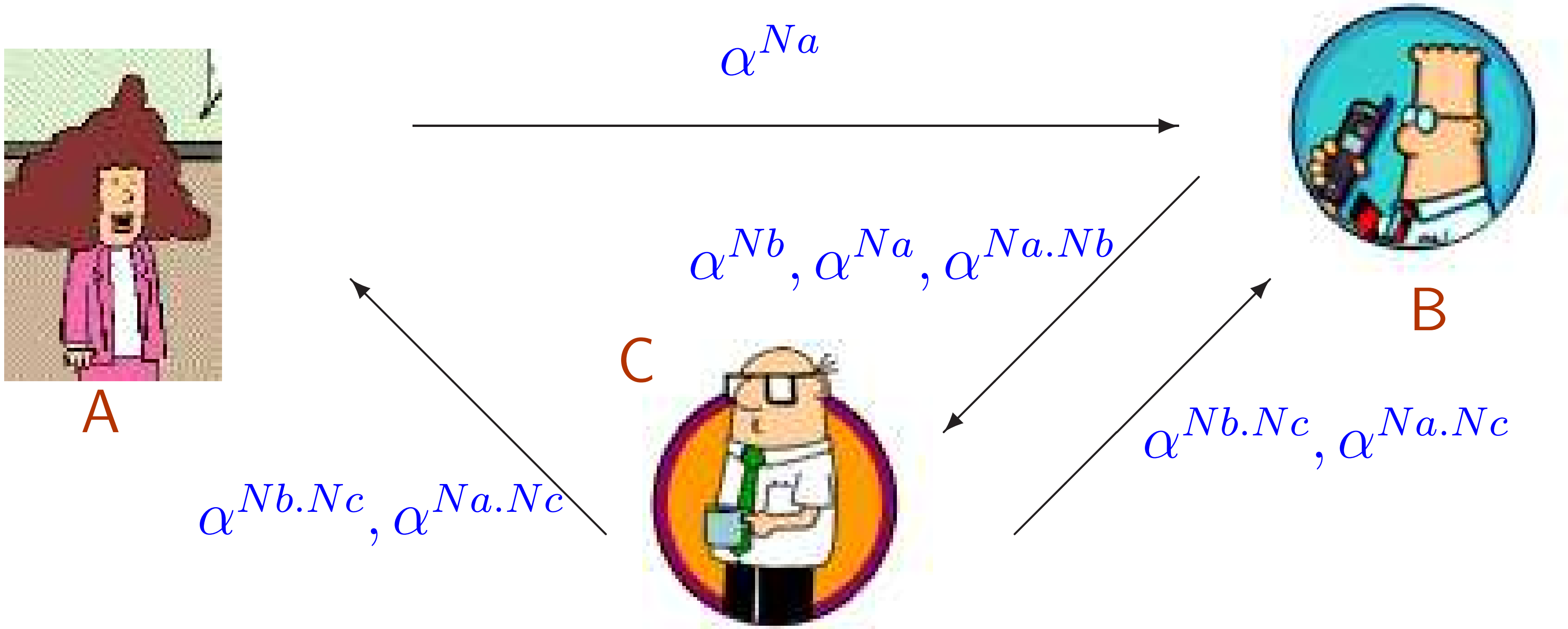



Group key $\alpha^{Na.Nb.Nc}$ is then computed by each participant

Code messages $\alpha^{x_1 \ldots x_n}$ by terms $e(x_1 + \ldots + x_n)$

$\Rightarrow +$ is $ACU$

# The ACU theory

| | |
|---|---|
| $x+(y+z)=(x+y)+z$ | Associativity |
| $x+y=y+x$ | Commutativity |
| $x+0=x$ | Unit |

# A protocol using XOR

An example protocol using XOR:

$+$ is XOR

Alice

Bob

## A protocol using XOR

An example protocol using XOR:

$+$ is XOR

Alice

$$\xrightarrow{\quad N_a + K_{ab} \quad}$$

Bob

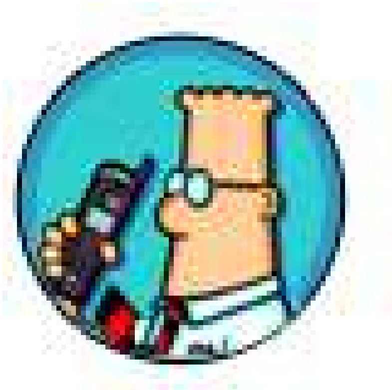
# A protocol using XOR

An example protocol using XOR:

$+$ is XOR

$$N_a + K_{ab} \longrightarrow$$

$$N_b + N_a \longleftarrow$$



Alice

Bob

## A protocol using XOR

An example protocol using XOR:

$+$ is XOR

Alice

$\xrightarrow{N_a + K_{ab}}$

$\xleftarrow{N_b + N_a}$

$\xrightarrow{S_{ab} + N_b}$

Bob

# A protocol using XOR

An example protocol using XOR:

$+$ is XOR

$$\text{Alice} \xrightarrow{N_a + K_{ab}} \text{Bob}$$

$$\text{Bob} \xrightarrow{N_b + N_a} \text{Alice}$$

$$\text{Alice} \xrightarrow{S_{ab} + N_b} \text{Bob}$$

Alice

Bob

Requires the XOR theory for modeling.

## The XOR theory

The ACU theory, together with the equation

$$x+x=0 \quad \text{Nilpotence}$$

## Another example: the Abelian Groups theory

The ACU theory, together with the equation

$$x+(-x)=0 \quad \text{Cancellation}$$

Typical equational theories that occur often in protocols are the ACU theory and its variants.