

Program Optimization

Exercise Sheet 1

Deadline: November 2, in 02.07.053, before 15:00.

Exercises marked with (P) are discussed and solved together at the lab sessions. Exercises marked with (H) are to be solved independently as homework.

Exercise 1: (P) VoTUM: Available expressions

No Points

Go to <http://www2.in.tum.de/projects/votum>, install VoTUM 0.7.5, and make yourself familiar with the system.

Now consider the following sequence of C statements:

```
int a,b,c;  
c = (a + b);  
c = (a - b);  
while (a + b < 5)  
    b = a + b;
```

Use VoTUM to compute available expressions and perform the optimization to eliminate redundant computations.

Exercise 2: (P) Semantics of Pre- and Post-increments

No Points

Assume we have a language \mathcal{L}_1 given by the following grammar:

$e ::= x$	Variable
c	Constant
$x++$	Post-increment of a variable
$++x$	Pre-increment of a variable
$e_1 + e_2$	Addition of two expressions
$e_1 - e_2$	Subtraction of two expressions

The pre- and post-increments should be familiar from Java (where $x++$ is written $\mathbf{x}++$). In the lecture, expressions had no side-effects, but these increments update the value of a given variable. Therefore, an expression now will both return a value and update the state $\rho \in Env = Var \rightarrow \mathbb{Z}$, so the semantics of an expression has the following type:

$$\llbracket e \rrbracket_1 : Env \rightarrow \mathbb{Z} \times Env.$$

Define for each kind of expression $e \in \mathcal{L}_1$, the semantic function $\llbracket e \rrbracket_1$. For this, and in the lecture, we use the update operation \oplus , which is defined as follows:

$$(\rho \oplus \{x \mapsto a\})(y) = \begin{cases} a & \text{if } y = x \\ \rho(y) & \text{otherwise.} \end{cases}$$

Exercise 3: (H) Translating Pre- and Post-increments

10 Points

Let \mathcal{L}_2 be the language defined by the following grammar:

$$\begin{aligned} l &::= e \mid x := e \mid l_1; l_2 \\ e &::= x \mid c \mid e_1 + e_2 \mid e_1 - e_2 \end{aligned}$$

Words generated by the first rule are essentially sequences of expressions and variable assignments. Right-hand sides of assignments are pure, i.e., they do not modify a state.

- a) Give the semantics for the language \mathcal{L}_2 . For that, define by structural induction a function of the type

$$\llbracket e \rrbracket_2: Env \rightarrow \mathbb{Z} \times Env.$$

The value of a sequence l is the value of the last expression in the sequence. For example,

$$\llbracket x := 5; x + 3 \rrbracket_2 \rho = \langle 8, \rho \oplus \{x \mapsto 5\} \rangle, \quad \llbracket 1; y := 3 \rrbracket_2 \rho = \langle 3, \rho \oplus \{y \mapsto 3\} \rangle.$$

- b) Define a function $\mathcal{T}: \mathcal{L}_1 \rightarrow \mathcal{L}_2$ which translates an expression of the language \mathcal{L}_1 to an expression of the language \mathcal{L}_2 that produces the same value and preserves the effect on variables occurring in the original expression (the translation may introduce temporary variables), i.e.,

$$\begin{aligned} \forall e \in \mathcal{L}_1, \forall \rho, \rho'_1, \rho'_2 \in Env, \forall v_1, v_2 \in \mathbb{Z}: \\ \langle v_1, \rho'_1 \rangle = \llbracket e \rrbracket_1 \rho \text{ and } \langle v_2, \rho'_2 \rangle = \llbracket \mathcal{T}(e) \rrbracket_2 \rho \text{ implies that} \\ v_1 = v_2 \text{ and } \forall x \in \mathbf{Vars}(e), \rho'_1 x = \rho'_2 x. \end{aligned}$$

For this, you should define the auxiliary function $\mathcal{T}': \mathcal{L}_1 \times \mathbb{Z} \rightarrow \mathcal{L}_2$. The integer parameter i indicates the next free temporary variable t_i , in which the result of the computation will be stored. For example,

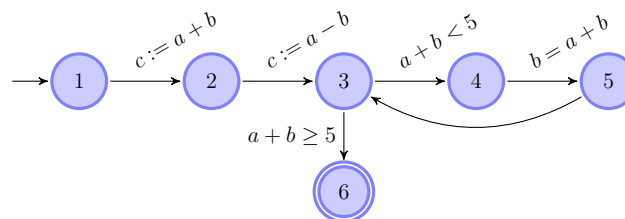
$$\begin{aligned} \mathcal{T}'(x \# + (y - \#x), 5) = \\ t_5 := x; x := x + 1; t_6 := y; x := x + 1; t_7 := x; t_6 := t_6 - t_7; t_5 := t_5 + t_6 \end{aligned}$$

We can then define $\mathcal{T}(e) = \mathcal{T}'(e, 1); t_1$. You do not *have to* give a proof of correctness, unless you really want to.

Exercise 4: (H) Computing Available Expressions

5 Points

The following is a control flow graph of the program from exercise 1.



Write down the constraint system and compute for each node u the set $A[u]$ of available expressions for this program.