Technische Universität München
Fakultät für Informatik
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Winter Semester 10/11
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Program Optimization

Exercise Sheet 2

Deadline: November 8, at the lecture.

Exercises marked with (P) are discussed and solved together at the lab sessions. Exercises marked with (H) are to be solved independently as homework.

Exercise 1: *(P/H) Lattices* *11 Points*

Recall definitions of a lattice $\mathbb{D} = \langle D, \sqsubseteq_{\mathbb{D}} \rangle$ and a complete lattice. Provide some examples of lattices and complete lattices. Let $\mathbb{D}_1, \mathbb{D}_2, \mathbb{D}$ be lattices.

a) (P) Define a product lattice $\mathbb{D}_1 \times \mathbb{D}_2$. Show that $\mathbb{D}_1 \times \mathbb{D}_2$ is a complete lattice if $\mathbb{D}_1, \mathbb{D}_2$ are complete lattices.
b) (H) Define a lattice of functions $X \to \mathbb{D}$, where X is a set. Show that $X \to \mathbb{D}$ is complete if $\mathbb{D}$ is complete.
c) (H) Define a lattice of monotonic functions $[\mathbb{D}_1 \to \mathbb{D}_2]$. Show that $[\mathbb{D}_1 \to \mathbb{D}_2]$ is complete if $\mathbb{D}_2$ is complete.

Exercise 2: *(H) Multiple constraints for a single variable* *4 Points*

Consider the following two constraint systems over a lattice $\mathbb{D}$:

$$(1) \quad \begin{array}{l} x \sqsupseteq f(x) \\ x \sqsupseteq g(x) \end{array} \qquad\qquad (2) \quad x \sqsupseteq f(x) \sqcup g(x)$$

where $f, g \colon \mathbb{D} \to \mathbb{D}$. Show that the following hold:

a) x is a solution of the constraint system (1) if and only if x is a solution of the constraint system (2);
b) x is the least solution of the constraint system (1) if and only if x is the least solution of the constraint system (2).