

Program Optimization

Exercise Sheet 5

Deadline: December 6, at the lecture.

Exercise 1: (P) *Fixpoint solving*

No Points

Recall the constraint system of exercise 4 (for Available Expressions analysis) from the Sheet 1 over the domain $\mathbb{D} = 2^{\{a+b, a-b\}}$:

$$\begin{aligned}\mathcal{A}[1] &\sqsupseteq \emptyset \\ \mathcal{A}[2] &\sqsupseteq \mathcal{A}[1] \cup \{a + b\} \\ \mathcal{A}[3] &\sqsupseteq \mathcal{A}[2] \cup \{a - b\} \\ \mathcal{A}[3] &\sqsupseteq \mathcal{A}[5] \\ \mathcal{A}[4] &\sqsupseteq \mathcal{A}[3] \cup \{a + b\} \\ \mathcal{A}[5] &\sqsupseteq \mathcal{A}[4] \setminus \{a + b, a - b\} \\ \mathcal{A}[6] &\sqsupseteq \mathcal{A}[3] \cup \{a + b\}\end{aligned}$$

Note that these have been written using the generic lattice operations, rather than the concrete set operations.

- Give the (vector-)function $F = \langle f_1, \dots, f_6 \rangle$ we use for fixpoint solving. For example, $f_1(x_1, \dots, x_6) = \emptyset$ and $f_2(x_1, \dots, x_6) = x_1 \cup \{a + b\}$.
- Using Kleene iteration, find the least fixpoint!

Exercise 2: (P) *Live Variables and Friends*

No Points

Go to <http://www.program-analysis.com> and try the PAG/WWW system. The four analyses, Available Expressions, Live Variables, Reaching Definitions, and Very Busy Expressions are data flow analyses that work on sets. Create a table indicating for each analysis: 1) Is it a backward or forward analysis? 2) What are the order and join operation?

Exercise 3: (H) *An Abstract Value Domain*

15 Points

- Define an abstract value domain \mathbb{D} that is ideally suited to answer, whether for an integer value x , the following hold: $x \geq 0$, $x < 0$, x is odd, and x is even. Use a bottom element to represent error values.
- Define the description relation $\Delta \subseteq \mathbb{Z} \times \mathbb{D}$.
- Define abstract arithmetic operations for multiplication $n \cdot m$ and integer exponentiation n^m (which returns error for $m < 0$).
- Show that your operations respect the description relation.
- Define the domain of a constant propagation analysis based on this value domain and specify the transformation function for the edge $x = e$.