Technische Universität München *Winter Semester 10/11*
Fakultät für Informatik *Vesal Vojdani*
Prof. Dr. H. Seidl *Aleksandr Karbyshev*

Program Optimization

Exercise Sheet 6

Deadline: December 13, at the lecture!

Exercise 1: *(H) Galois Connections* *12 Points*

A Galois connection between lattices $\mathbb{D}_1$ and $\mathbb{D}_2$ consists of two functions $\alpha\colon \mathbb{D}_1 \to \mathbb{D}_2$ and $\gamma\colon \mathbb{D}_2 \to \mathbb{D}_1$, such that $\alpha(x) \sqsubseteq y$ iff $x \sqsubseteq \gamma(y)$, for all $x \in \mathbb{D}_1$, $y \in \mathbb{D}_2$.

a) Show that α and γ are monotonic.

b) Consider a Galois connection (α, γ) between the powerset lattice $\mathcal{P}(X)$ (for a set X) and a lattice $\mathbb{D}$. We define the relation: $x \Delta d \iff x \in \gamma(d)$. Show that Δ is a description relation, i.e., $x \Delta a_1 \wedge a_1 \sqsubseteq a_2 \implies x \Delta a_2$.

c) Show that for a Galois connection, α is uniquely determined through γ:

$$\alpha(d_1) = \bigsqcap \{d_2 \in \mathbb{D}_2 \mid \gamma(d_2) \sqsupseteq d_1\}$$

d) Give an example of a description relation $\Delta \subseteq X \times \mathbb{D}$, such that the concretization obtained by taking $\gamma(d) = \{x \mid x \Delta d\}$ is not a Galois connection between the powerset lattice $\mathcal{P}(X)$ and the lattice $\mathbb{D}$.

Exercise 2: *Widening and Narrowing* *5 Points*

Recall that for widenings, instead of solving $x \sqsupseteq f(x)$, we solve the equation $x = x \sqcup f(x)$ by iterating the accumulating function $g(x) = x \sqcup f(x)$. We can then apply widenings: $g'(x) = x \sqcup\!\!\!\!\sqcup f(x)$. This is safe, even if f is not monotonic.

When we use accelerated narrowing, we similarly define $h(x) = x \sqcap f(x)$, and apply the narrowing $h'(x) = x \sqcap\!\!\!\!\sqcap f(x)$, but now monotonicity is crucial. *Why?* What can go wrong, if f is not monotonic? Why is the situation different from the case with widenings?