Discussion:

→ Caveat: Widening also returns for non-monotonic f_i a solution. Narrowing is only applicable to monotonic f_i !!

→ In the example, accelerated narrowing already returns the optimal result :-)

→ If the operator $\sqcap\!\!\sqcap$ only allows for finitely many improvements of values, we may execute narrowing until stabilization.

→ In case of interval analysis these are at most:

$$\#points \cdot (1 + 2 \cdot \#Vars)$$

1.6 Pointer Analysis

Questions:

- $\rightarrow$ Are two addresses possibly equal?
- $\rightarrow$ Are two addresses definitively equal?

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$\rightarrow$ Are two addresses definitively equal? Must Alias

$\Longrightarrow$ Alias Analysis

The analyses so far without alias information:

(1) Available Expressions:

- Extend the set *Expr* of expressions by occurring loads $M[e]$.

- Extend the Effects of Edges:

$$
\begin{array}{lll}
[\![x = e;]\!]^\sharp A & = & (A \cup \{e\}) \backslash Expr_x \\
[\![x = M[e];]\!]^\sharp A & = & (A \cup \{e, M[e]\}) \backslash Expr_x \\
[\![M[e_1] = e_2;]\!]^\sharp A & = & (A \cup \{e_1, e_2\}) \backslash Loads
\end{array}
$$

(2) Values of Variables:

- Extend the set *Expr* of expressions by occurring loads $M[e]$.

- Extend the Effects of Edges:

$$[\![x = M[e];]\!]^\sharp\, V\, e' \quad = \quad \begin{cases} \{x\} & \text{if} \quad e' = M[e] \\ \emptyset & \text{if} \quad e' = e \\ V\, e' \backslash \{x\} & \text{otherwise} \end{cases}$$

$$[\![M[e_1] = e_2;]\!]^\sharp\, V\, e' \quad = \quad \begin{cases} \emptyset & \text{if} \quad e' \in \{e_1, e_2\} \\ V\, e' & \text{otherwise} \end{cases}$$

(3) Constant Propagation:

- Extend the abstract state by an abstract store M

- Execute accesses to known memory locations!

$$[\![x = M[e];]\!]^\sharp\,(D, M) \quad = \quad \begin{cases} (D \oplus \{x \mapsto M\,a\}, M) & \text{if} \\ & [\![e]\!]^\sharp\, D = a \sqsubset \top \\ (D \oplus \{x \mapsto \top\}, M) & \text{otherwise} \end{cases}$$

$$[\![M[e_1] = e_2;]\!]^\sharp\,(D, M) \quad = \quad \begin{cases} (D, M \oplus \{a \mapsto [\![e_2]\!]^\sharp D\}) & \text{if} \\ & [\![e_1]\!]^\sharp\, D = a \sqsubset \top \\ (D, \underline{\top}) & \text{otherwise} \end{cases} \qquad \text{where}$$

$$\underline{\top}\, a \quad = \quad \top \qquad\qquad (a \in \mathbb{N})$$

Problems:

- Addresses are from $\mathbb{N}$:-(

  There are no infinite strictly ascending chains, but ...
- Exact addresses at compile-time are rarely known :-(
- At the same program point, typically different addresses are accessed ...
- Storing at an unknown address destroys all information M :-(

$\Longrightarrow$ constant propagation fails :-(

$\Longrightarrow$ memory accesses/pointers kill precision :-(

Simplification:

- We consider pointers to the beginning of blocks A which allow indexed accesses $A[i]$:-)
- We ignore well-typedness of the blocks.
- New statements:

$$x = \mathsf{new}(); \quad // \quad \text{allocation of a new block}$$

$$x = y[e]; \quad // \quad \text{indexed read access to a block}$$

$$y[e_1] = e_2; \quad // \quad \text{indexed write access to a block}$$

- Blocks are possibly infinite :-)
- For simplicity, all pointers point to the beginning of a block.

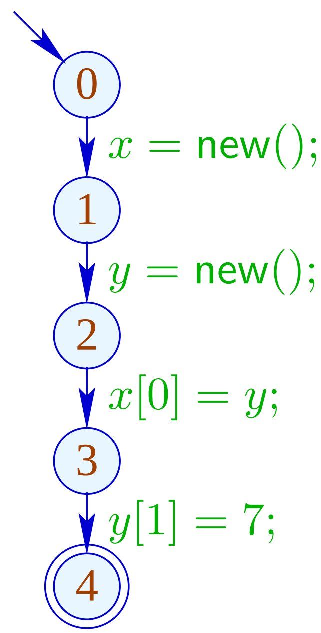
Simple Example:

$$x = \mathsf{new}();$$

$$y = \mathsf{new}();$$

$$x[0] = y;$$

$$y[1] = 7;$$



The Semantics:

x	
y	

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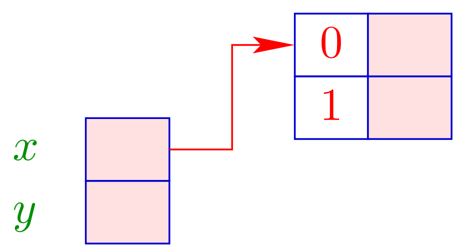

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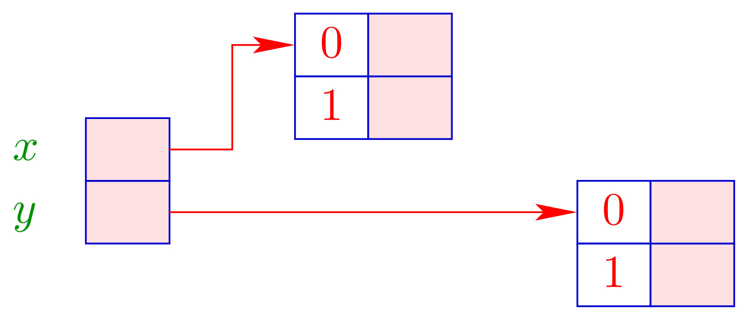

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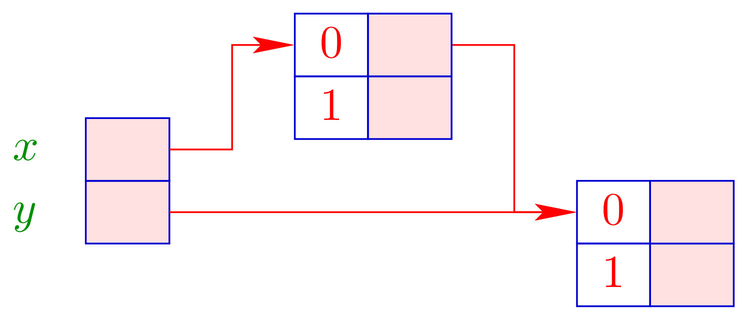

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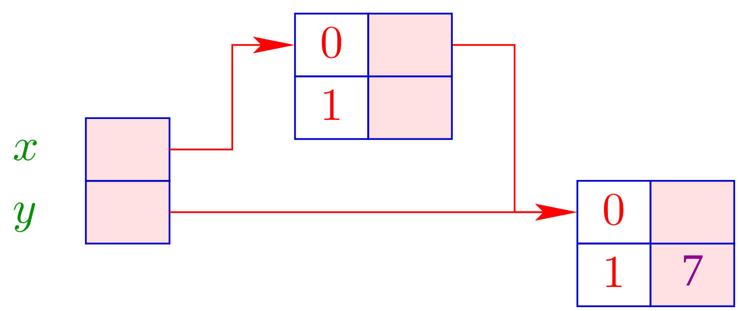


More Complex Example:

$r = \mathsf{Null};$

$\mathsf{while}\ (t \neq \mathsf{Null})\ \{$

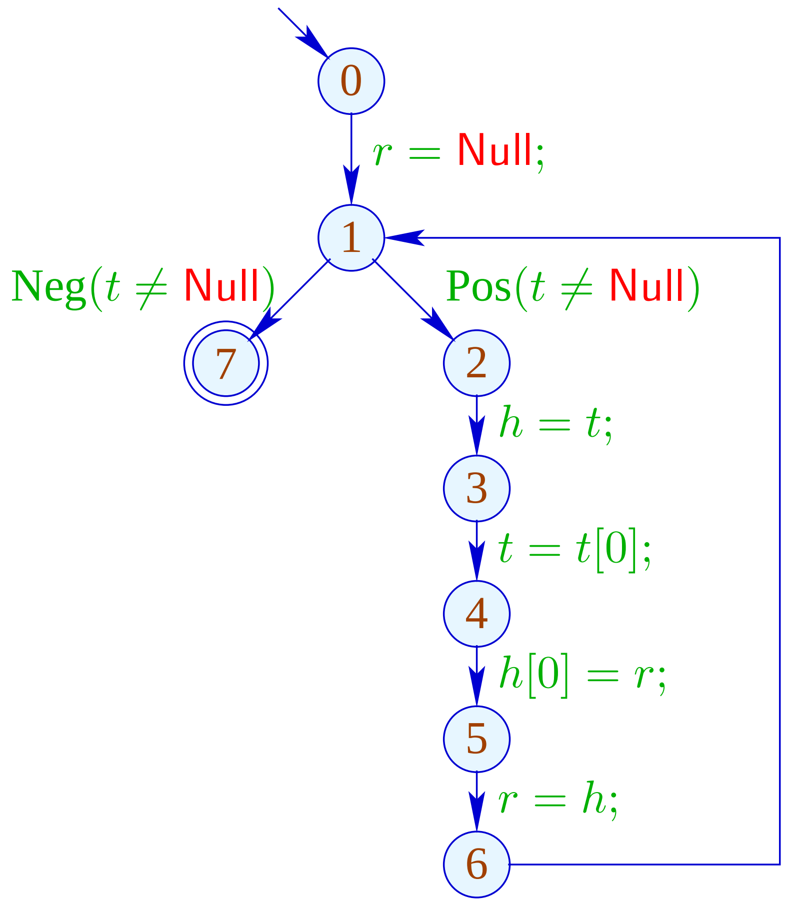
$\quad h = t;$

$\quad t = t[0];$

$\quad h[0] = r;$

$\quad r = h;$

$\}$



Concrete Semantics:

A store consists of a finite collection of blocks.

After h new-operations we obtain:

$$
\begin{array}{llll}
\mathit{Addr}_h & = & \{\mathsf{ref}\ a \mid 0 \leq a < h\} & // \quad \text{addresses} \\
\mathit{Val}_h & = & \mathit{Addr}_h \cup \mathbb{Z} & // \quad \text{values} \\
\mathit{Store}_h & = & (\mathit{Addr}_h \times \mathbb{N}_0) \to \mathit{Val}_h & // \quad \text{store} \\
\mathit{State}_h & = & (\mathit{Vars} \to \mathit{Val}_h) \times \mathit{Store}_h & // \quad \text{states}
\end{array}
$$

For simplicity, we set: $0 = \mathsf{Null}$

Let $(\rho, \mu) \in State_h$. Then we obtain for the new edges:

$$\begin{array}{lll} [\![x = \mathsf{new}();]\!] \, (\rho, \mu) & = & (\rho \oplus \{x \mapsto \mathsf{ref}\; h\}, \\ & & \;\mu \oplus \{(\mathsf{ref}\; h, i) \mapsto 0 \mid i \in \mathbb{N}_0\}) \\ [\![x = y[e];]\!] \, (\rho, \mu) & = & (\rho \oplus \{x \mapsto \mu\,(\rho\, y, [\![e]\!]\, \rho)\}, \mu) \\ [\![y[e_1] = e_2;]\!] \, (\rho, \mu) & = & (\rho, \mu \oplus \{(\rho\, y, [\![e_1]\!]\, \rho) \mapsto [\![e_2]\!]\, \rho\}) \end{array}$$

Caveat:

This semantics is too detailled in that it computes with absolute Addresses. Accordingly, the two programs:

$$\begin{array}{lcl} x = \mathsf{new}(); & \qquad\qquad & y = \mathsf{new}(); \\ y = \mathsf{new}(); & & x = \mathsf{new}(); \end{array}$$

are not considered as equivalent !!?

Possible Solution:

Define equivalence only up to permutation of addresses :-)

Alias Analysis 1. Idea:

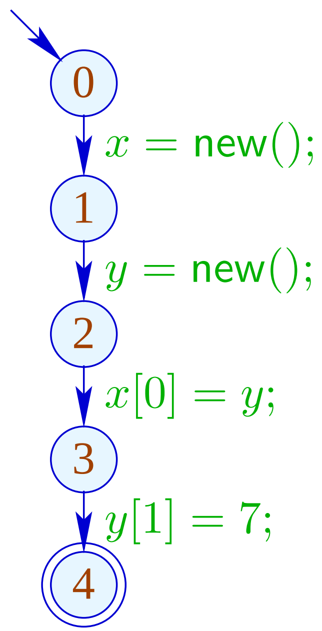
- Distinguish finitely many classes of blocks.
- Collect all addresses of a block into one set!
- Use sets of addresses as abstract values!

$\Longrightarrow$ Points-to-Analysis

$$
\begin{array}{lcll}
\mathit{Addr}^\sharp & = & \mathit{Edges} & // \text{ creation edges} \\
\mathit{Val}^\sharp & = & 2^{\mathit{Addr}^\sharp} & // \text{ abstract values} \\
\mathit{Store}^\sharp & = & \mathit{Addr}^\sharp \to \mathit{Val}^\sharp & // \text{ abstract store} \\
\mathit{State}^\sharp & = & (\mathit{Vars} \to \mathit{Val}^\sharp) \times \mathit{Store}^\sharp & // \text{ abstract states}
\end{array}
$$

// complete lattice !!!

... in the Simple Example:



	x	y	$(0,1)$
0	$\emptyset$	$\emptyset$	$\emptyset$
1	$\{(0,1)\}$	$\emptyset$	$\emptyset$
2	$\{(0,1)\}$	$\{(1,2)\}$	$\emptyset$
3	$\{(0,1)\}$	$\{(1,2)\}$	$\{(1,2)\}$
4	$\{(0,1)\}$	$\{(1,2)\}$	$\{(1,2)\}$

The Effects of Edges:

$$
\begin{array}{lcll}
[\![(\_, ;, \_)]\!]^\sharp\,(D, M) & = & (D, M) & \\
[\![(\_, \mathrm{Pos}(e), \_)]\!]^\sharp\,(D, M) & = & (D, M) & \\
[\![(\_, x = y;, \_)]\!]^\sharp\,(D, M) & = & (D \oplus \{x \mapsto D\,y\}, M) & \\
[\![(\_, x = e;, \_)]\!]^\sharp\,(D, M) & = & (D \oplus \{x \mapsto \emptyset\}, M) \qquad , & e \notin \mathit{Vars} \\
\\
[\![(u, x = \mathsf{new}();, v)]\!]^\sharp\,(D, M) & = & (D \oplus \{x \mapsto \{(u, v)\}\}, M) & \\
[\![(\_, x = y[e];, \_)]\!]^\sharp\,(D, M) & = & (D \oplus \{x \mapsto \bigcup\{M(f) \mid f \in D\,y\}\}, M) & \\
[\![(\_, y[e_1] = x;, \_)]\!]^\sharp\,(D, M) & = & (D, M \oplus \{f \mapsto (M\,f \cup D\,x) \mid f \in D\,y\}) &
\end{array}
$$

Caveat:

- The value Null has been ignored. Dereferencing of Null or negative indices are not detected :-(
- Destructive updates are only possible for variables, not for blocks in storage!

  $\Longrightarrow$ no information, if not all block entries are initialized before use :-((
- The effects now depend on the edge itself.

  The analysis cannot be proven correct w.r.t. the reference semantics :-(

  In order to prove correctness, we first instrument the concrete semantics with extra information which records where a block has been created.

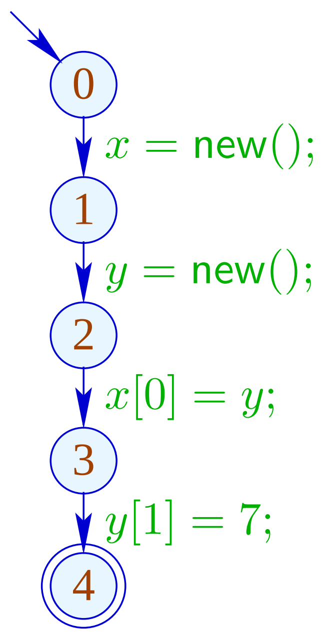
...

- We compute possible points-to information.
- From that, we can extract may-alias information.
- The analysis can be rather expensive — without finding very much :-(
- Separate information for each program point can perhaps be abandoned ??

2. Idea:

Compute for each variable and address a value which safely approximates the values at every program point simultaneously !

... in the Simple Example:



x	$\{(0, 1)\}$
y	$\{(1, 2)\}$
$(0, 1)$	$\{(1, 2)\}$
$(1, 2)$	$\emptyset$

Each edge (u, lab, v) gives rise to constraints:

lab	$Constraint$
$x = y;$	$\mathcal{P}[x] \supseteq \mathcal{P}[y]$
$x = \mathsf{new}();$	$\mathcal{P}[x] \supseteq \{(u, v)\}$
$x = y[e];$	$\mathcal{P}[x] \supseteq \bigcup\{\mathcal{P}[f] \mid f \in \mathcal{P}[y]\}$
$y[e_1] = x;$	$\mathcal{P}[f] \supseteq (f \in \mathcal{P}[y])\,?\,\mathcal{P}[x] : \emptyset$
	for all $f \in Addr^\sharp$

Other edges have no effect :-)