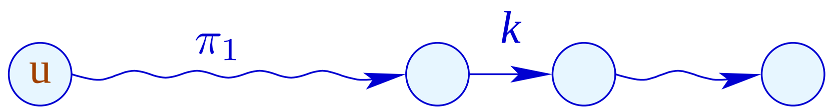
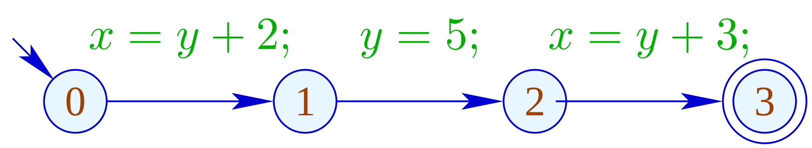


Thereby, the set of all defined or used variables at an edge
$k = (\_, lab, \_)$ is defined by:

lab	used	defined
$;$	$\emptyset$	$\emptyset$
$\text{Pos}\,(e)$	$\mathit{Vars}\,(e)$	$\emptyset$
$\text{Neg}\,(e)$	$\mathit{Vars}\,(e)$	$\emptyset$
$x = e;$	$\mathit{Vars}\,(e)$	$\{x\}$
$x = M[e];$	$\mathit{Vars}\,(e)$	$\{x\}$
$M[e_1] = e_2;$	$\mathit{Vars}\,(e_1) \cup \mathit{Vars}\,(e_2)$	$\emptyset$

A variable x which is not live at u along π (relative to X) is called dead at u along π (relative to X).

Example:



where $X = \emptyset$. Then we observe:

	live	dead
0	$\{y\}$	$\{x\}$
1	$\emptyset$	$\{x, y\}$
2	$\{y\}$	$\{x\}$
3	$\emptyset$	$\{x, y\}$

The variable x is live at u (relative to X) if x is live at u along some path to the exit (relative to X). Otherwise, x is called dead at u (relative to X).

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Question:

How can the sets of all dead/live variables be computed for every u ???

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Question:

How can the sets of all dead/live variables be computed for every u ???

Idea:

For every edge $k = (u, \_, v)$, define a function $[\![k]\!]^\sharp$ which transforms the set of variables which are live at v into the set of variables which are live at u ...

Let $\mathbb{L} = 2^{Vars}$.

For $k = (\_, lab, \_)$, define $[\![k]\!]^\sharp = [\![lab]\!]^\sharp$ by:

$$
\begin{array}{lll}
[\![;]\!]^\sharp\, L & = & L \\
[\![\mathrm{Pos}(e)]\!]^\sharp\, L & = & [\![\mathrm{Neg}(e)]\!]^\sharp\, L \quad = \quad L \cup Vars(e) \\
[\![x = e;]\!]^\sharp\, L & = & (L \backslash \{x\}) \cup Vars(e) \\
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[\![M[e_1] = e_2;]\!]^\sharp\, L & = & L \cup Vars(e_1) \cup Vars(e_2)
\end{array}
$$

Let $\quad \mathbb{L} = 2^{\mathit{Vars}}$.

For $\quad k = (\_, \mathit{lab}, \_)$, define $\quad [\![k]\!]^\sharp = [\![\mathit{lab}]\!]^\sharp \quad$ by:

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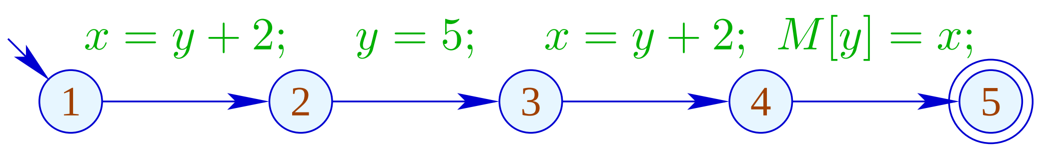
[\![M[e_1] = e_2;]\!]^\sharp\, L & = & L \cup \mathit{Vars}(e_1) \cup \mathit{Vars}(e_2)
\end{array}
$$

$[\![k]\!]^\sharp \quad$ can again be composed to the effects of $\quad [\![\pi]\!]^\sharp \quad$ of paths $\pi = k_1 \ldots k_r \quad$ by:

$$[\![\pi]\!]^\sharp = [\![k_1]\!]^\sharp \circ \ldots \circ [\![k_r]\!]^\sharp$$

We verify that these definitions are meaningful :-)



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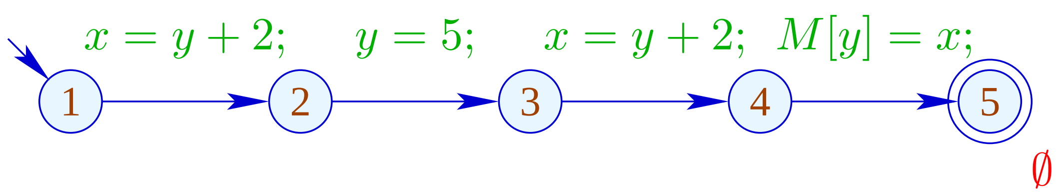

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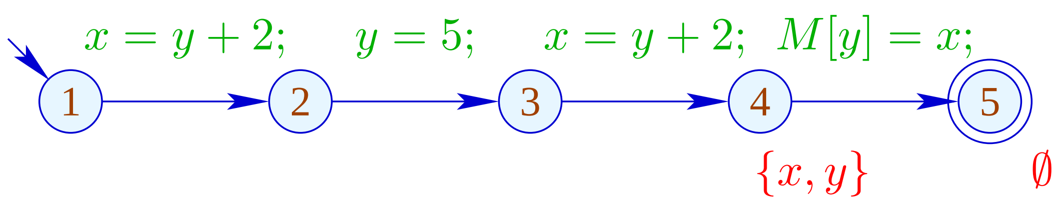

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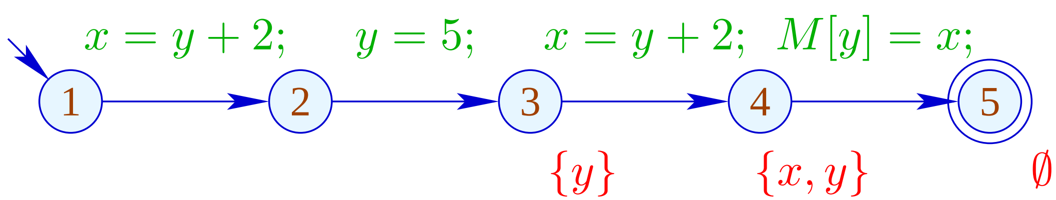

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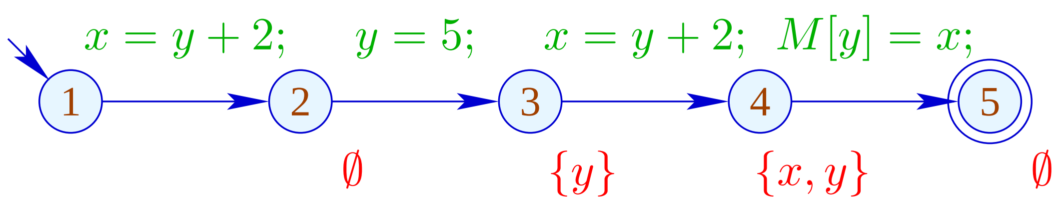

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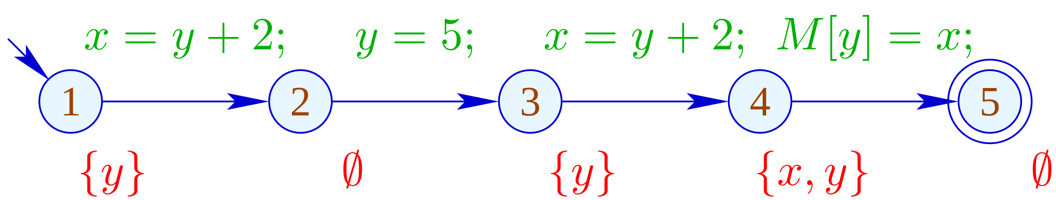


The set of variables which are live at u then is given by:

$$\mathcal{L}^*[u] = \bigcup\{[\![\pi]\!]^\sharp X \mid \pi : u \to^* stop\}$$

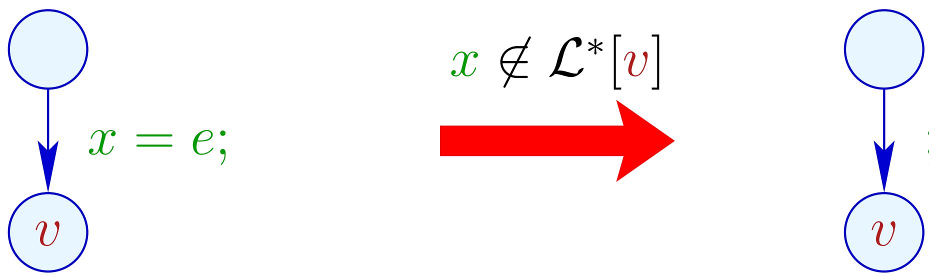
... literally:

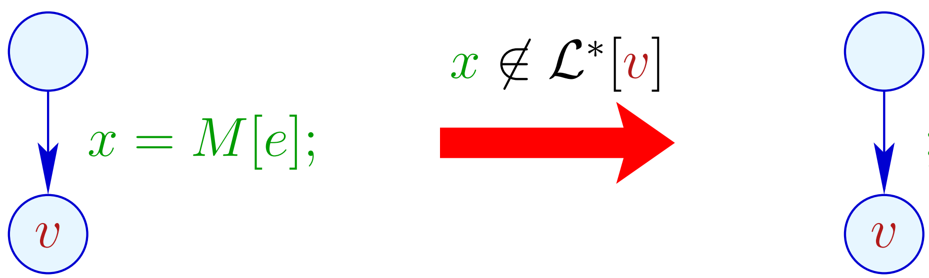
- The paths start in u :-)

  $\Longrightarrow$ As partial ordering for $\mathbb{L}$ we use $\sqsubseteq = \subseteq$.

- The set of variables which are live at program exit is given by the set X :-)

Transformation 2:





Correctness Proof:

- $\rightarrow$ Correctness of the effects of edges: If L is the set of variables which are live at the exit of the path π , then $[\![\pi]\!]^\sharp\, L$ is the set of variables which are live at the beginning of π :-)
- $\rightarrow$ Correctness of the transformation along a path: If the value of a variable is accessed, this variable is necessarily live. The value of dead variables thus is irrelevant :-)
- $\rightarrow$ Correctness of the transformation: In any execution of the transformed programs, the live variables always receive the same values :-))

Computation of the sets $\mathcal{L}^*[u]$:

(1) Collecting constraints:

$$\begin{array}{lll} \mathcal{L}[stop] & \supseteq & X \\ \mathcal{L}[u] & \supseteq & [\![k]\!]^\sharp\,(\mathcal{L}[v]) \qquad k = (u, \_, v) \quad \text{edge} \end{array}$$

(2) Solving the constraint system by means of RR iteration.

Since $\mathbb{L}$ is finite, the iteration will terminate :-)

(3) If the exit is (formally) reachable from every program point, then the smallest solution $\mathcal{L}$ of the constraint system equals $\mathcal{L}^*$ since all $[\![k]\!]^\sharp$ are distributive :-))

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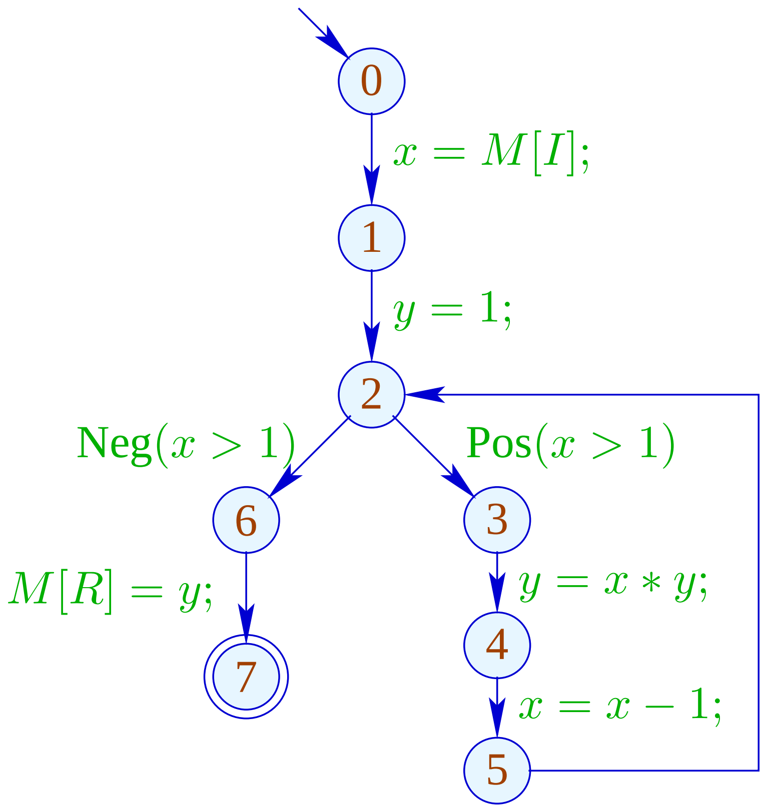
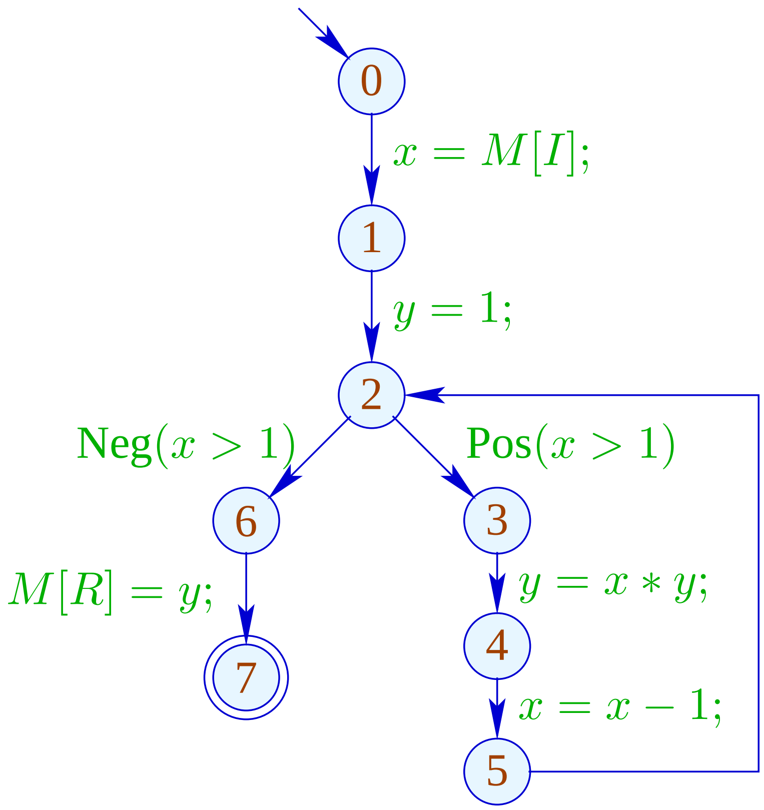
Caveat: The information is propagated backwards !!!

Example:



$$
\begin{array}{lll}
\mathcal{L}[0] & \supseteq & (\mathcal{L}[1]\backslash\{x\}) \cup \{I\} \\
\mathcal{L}[1] & \supseteq & \mathcal{L}[2]\backslash\{y\} \\
\mathcal{L}[2] & \supseteq & (\mathcal{L}[6] \cup \{x\}) \cup (\mathcal{L}[3] \cup \{x\}) \\
\mathcal{L}[3] & \supseteq & (\mathcal{L}[4]\backslash\{y\}) \cup \{x, y\} \\
\mathcal{L}[4] & \supseteq & (\mathcal{L}[5]\backslash\{x\}) \cup \{x\} \\
\mathcal{L}[5] & \supseteq & \mathcal{L}[2] \\
\mathcal{L}[6] & \supseteq & \mathcal{L}[7] \cup \{y, R\} \\
\mathcal{L}[7] & \supseteq & \emptyset
\end{array}
$$

Example:

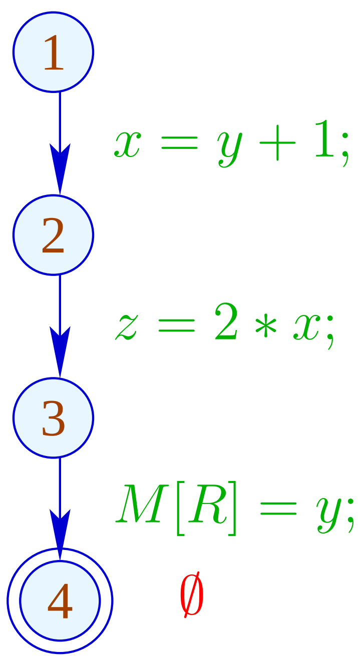


	1	2
7	$\emptyset$	
6	$\{y, R\}$	
2	$\{x, y, R\}$	dito
5	$\{x, y, R\}$	
4	$\{x, y, R\}$	
3	$\{x, y, R\}$	
1	$\{x, R\}$	
0	$\{I, R\}$	

The left-hand side of no assignment is dead :-)

Caveat:

Removal of assignments to dead variables may kill further variables:



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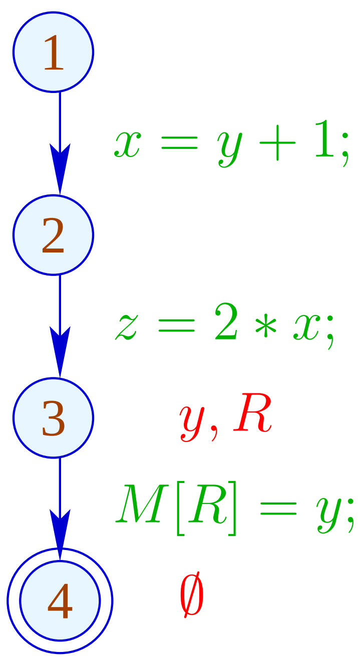

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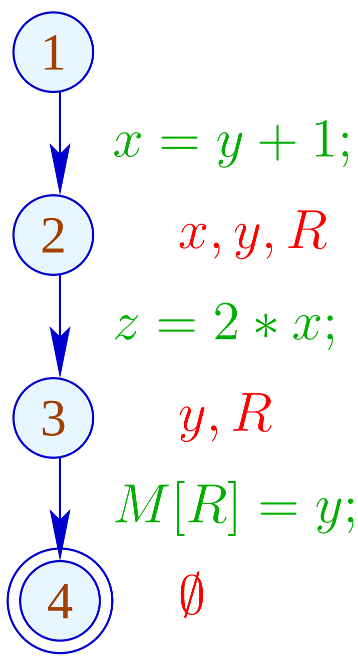

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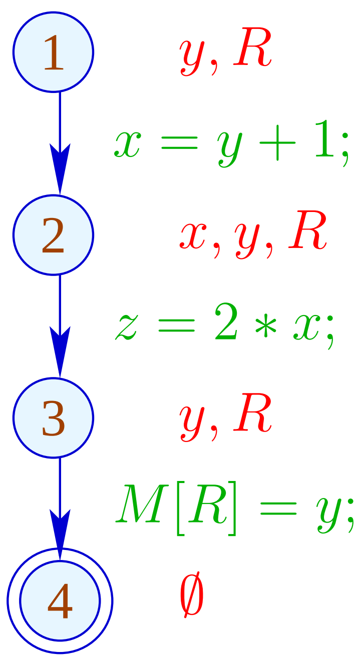

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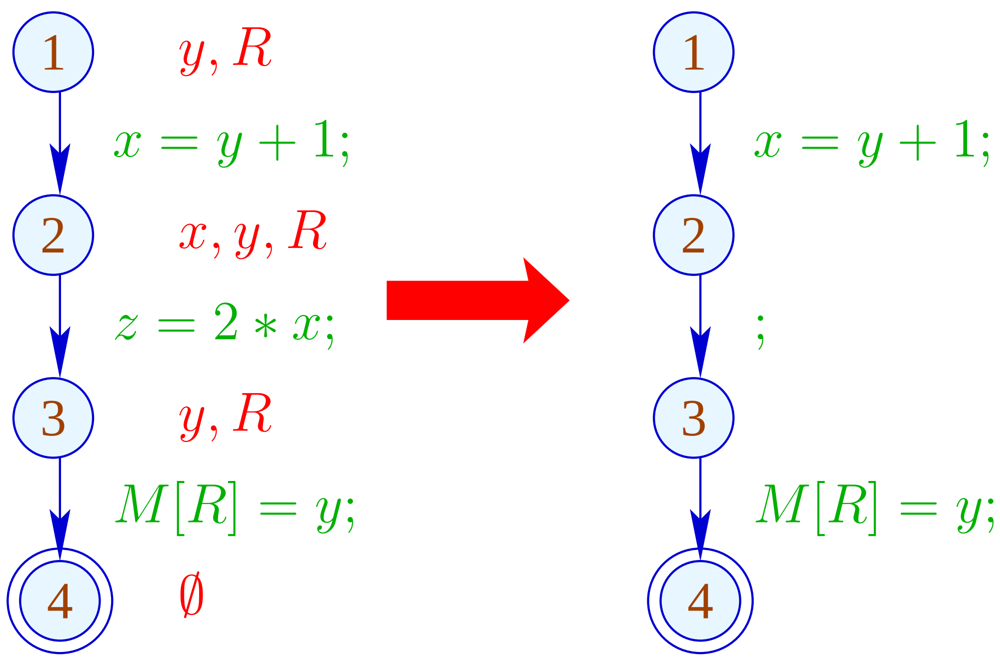

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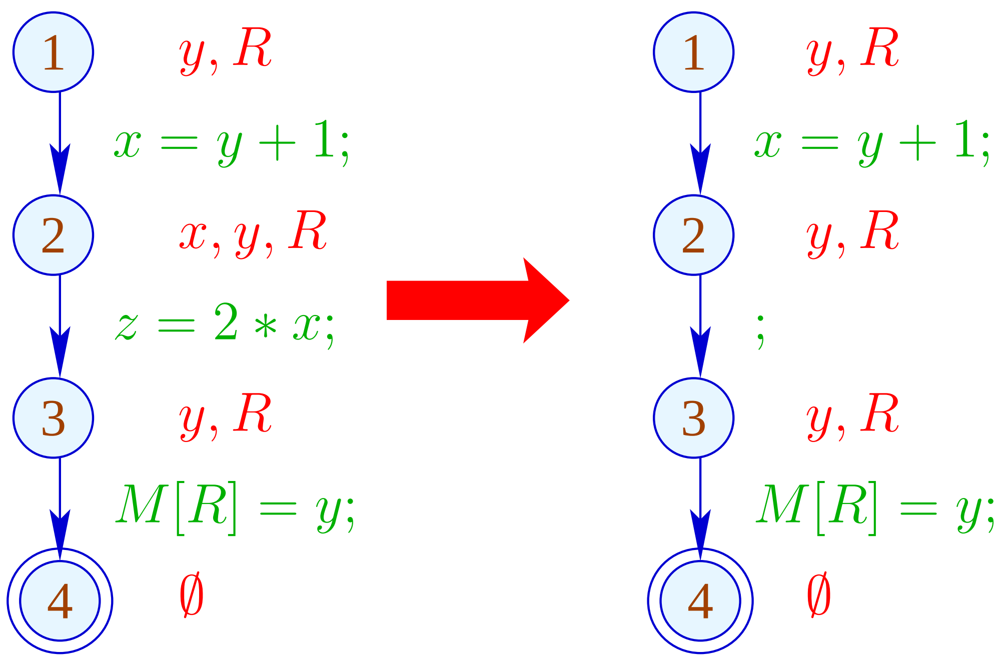

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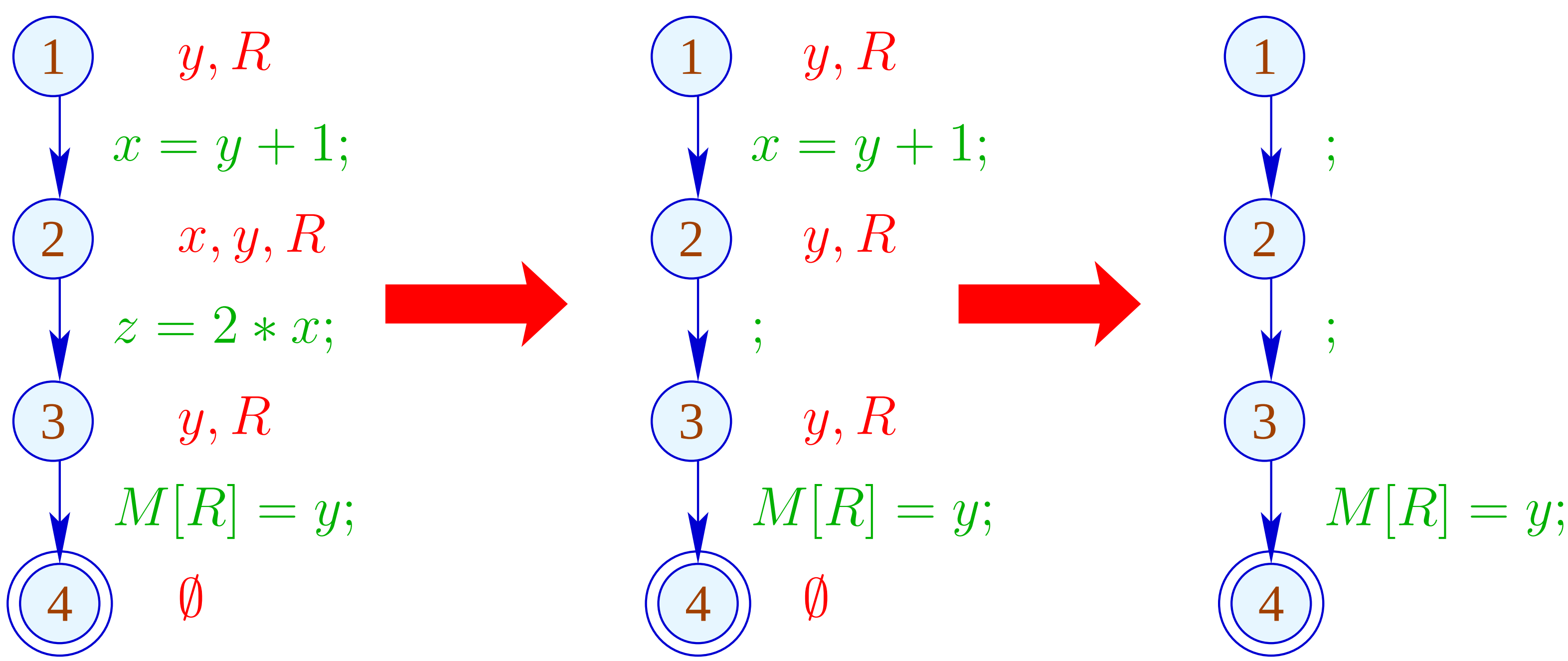


Re-analyzing the program is inconvenient :-(

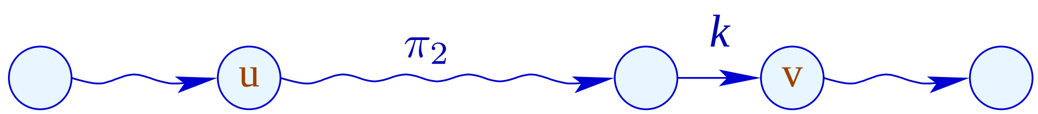
Idea: Analyze true liveness!

x is called truely live at u along a path π (relative to X), either

if $x \in X$, π does not contain a definition of x; or

if π can be decomposed into $\pi = \pi_1\, k\, \pi_2$ such that:

- k is a true use of x ;
- π_1 does not contain any definition of x.

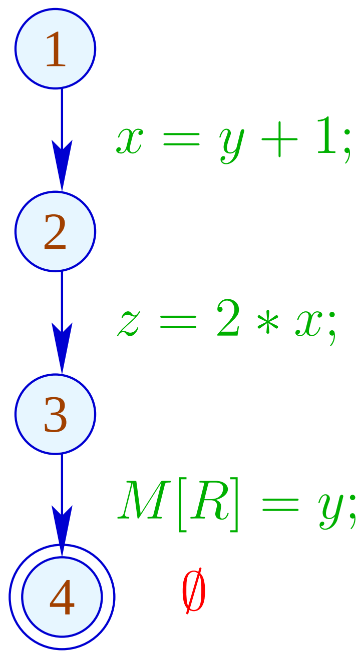


The set of truely used variables at an edge $k = (\_, lab, v)$ is defined as:

lab	truely used
$;$	$\emptyset$
$\mathrm{Pos}\,(e)$	$Vars\,(e)$
$\mathrm{Neg}\,(e)$	$Vars\,(e)$
$x = e;$	$Vars\,(e)$ $(*)$
$x = M[e];$	$Vars\,(e)$ $(*)$
$M[e_1] = e_2;$	$Vars(e_1) \cup Vars(e_2)$

$(*)$ – given that x is truely live at v :-)

Example:



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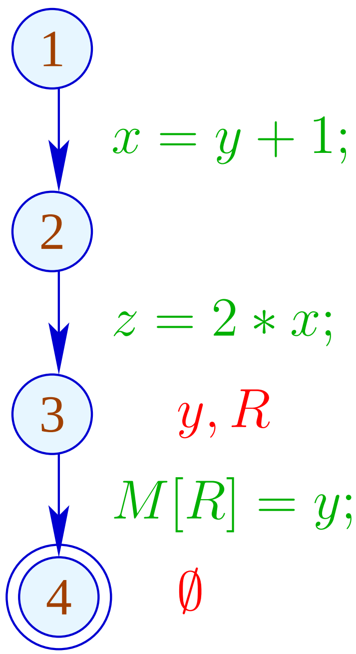

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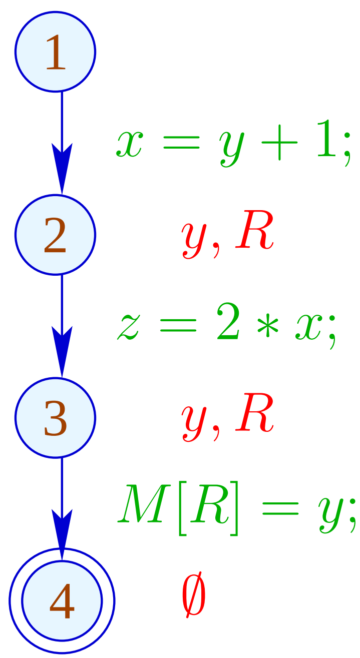

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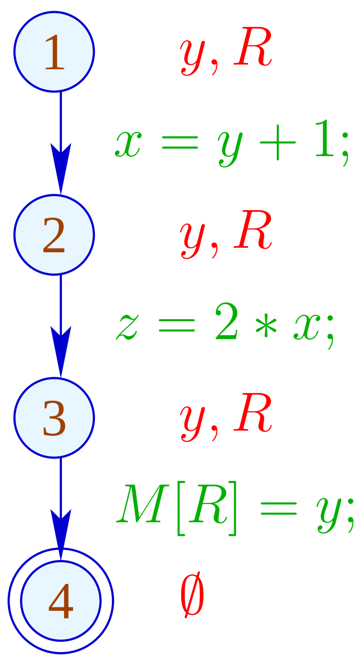

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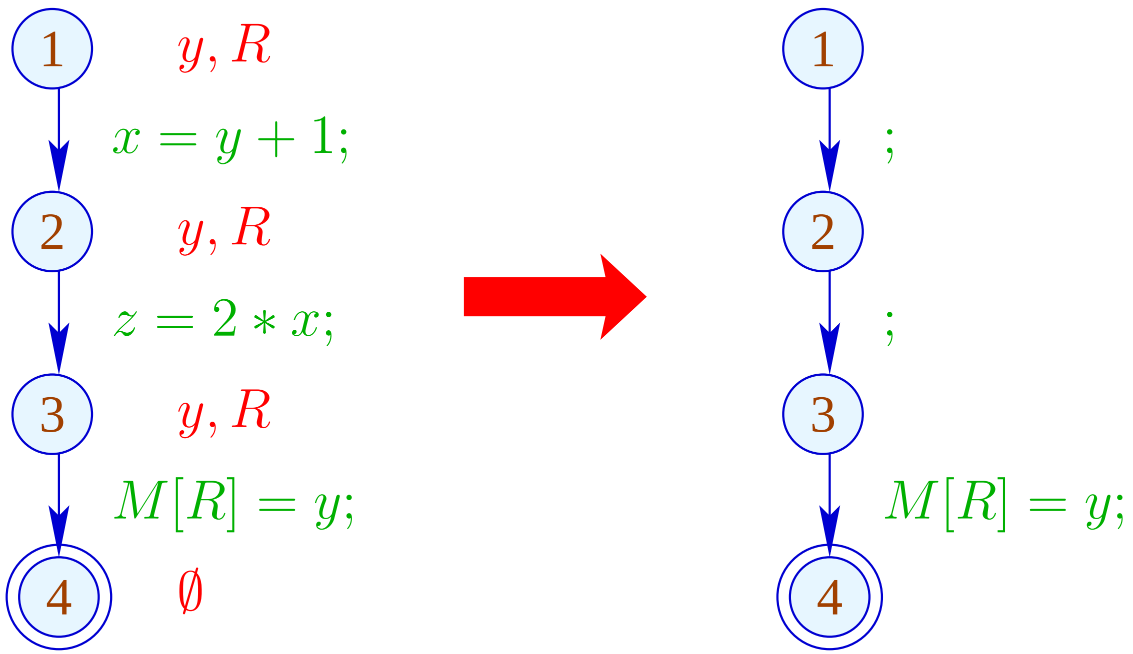

The Effects of Edges:

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[\![;]\!]^\sharp \, L & = & L \\
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[\![x = e;]\!]^\sharp \, L & = & (L \backslash \{x\}) \cup \; (x \in L) \, ? \, \mathit{Vars}(e) : \; \emptyset \\
[\![x = M[e];]\!]^\sharp \, L & = & (L \backslash \{x\}) \cup \; (x \in L) \, ? \, \mathit{Vars}(e) : \; \emptyset \\
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\end{array}
$$

Note:

- The effects of edges for truely live variables are more complicated than for live variables :-)
- Nonetheless, they are distributive !!

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  To see this, consider for $\mathbb{D} = 2^U$, $f\,y = (u \in y)\,?\,b :\ \emptyset$ We verify:

$$\begin{array}{lll} f\,(y_1 \cup y_2) & = & (u \in y_1 \cup y_2)\,?\,b :\ \emptyset \\ & = & (u \in y_1 \vee u \in y_2)\,?\,b :\ \emptyset \\ & = & (u \in y_1)\,?\,b :\ \emptyset \cup (u \in y_2)\,?\,b :\ \emptyset \\ & = & f\,y_1 \cup f\,y_2 \end{array}$$

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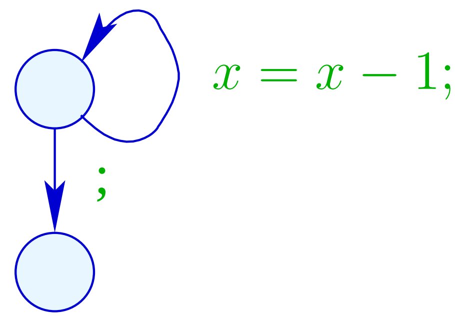
\end{array}
$$

$\Longrightarrow$ the constraint system yields the MOP :-))

- True liveness detects more superfluous assignments than repeated liveness !!!



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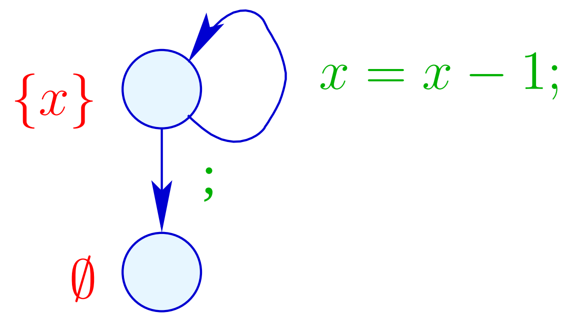
Liveness:



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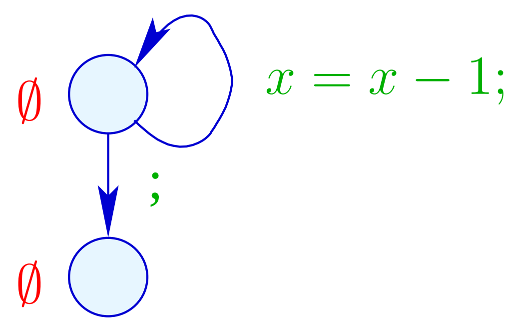
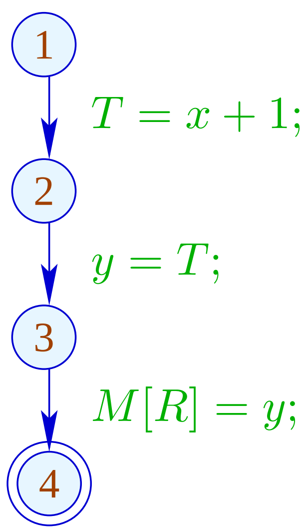
True Liveness:



1.3 Removing Superfluous Moves

Example:



This variable-variable assignment is obviously useless :-(

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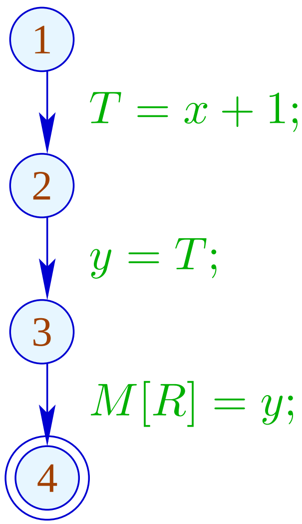

This variable-variable assignment is obviously useless :-(

Instead of y, we could also store T :-)

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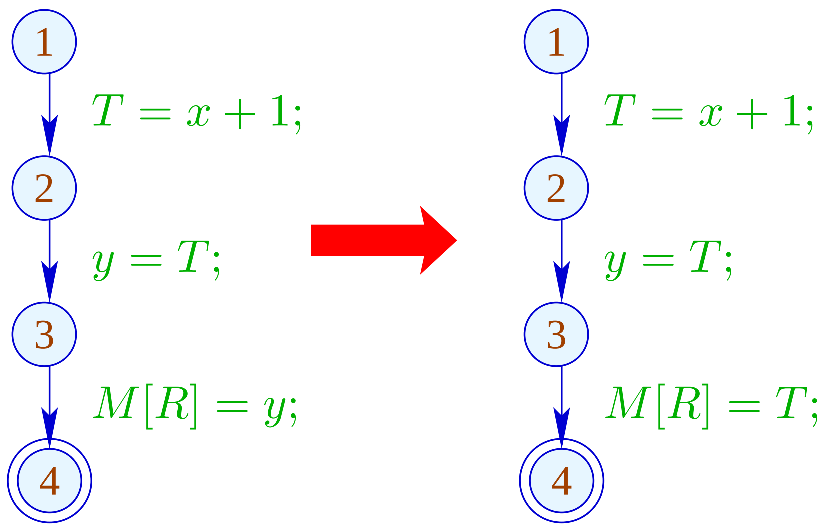

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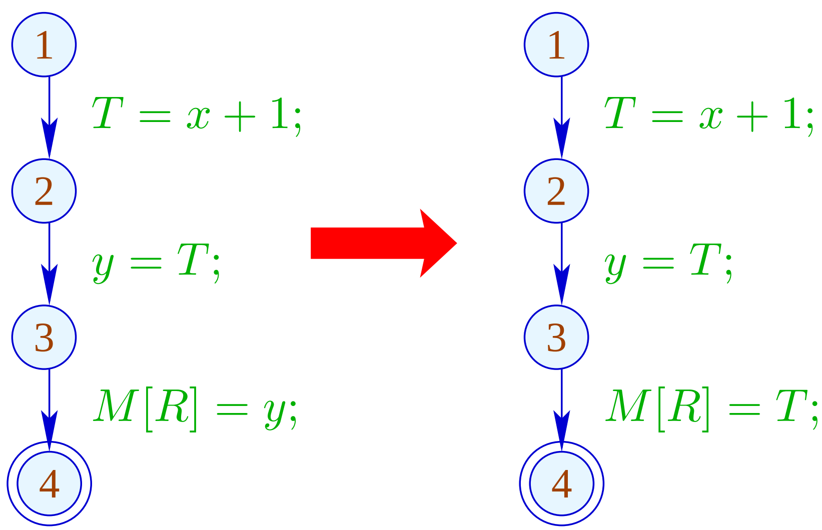

Advantage: Now, y has become dead :-))

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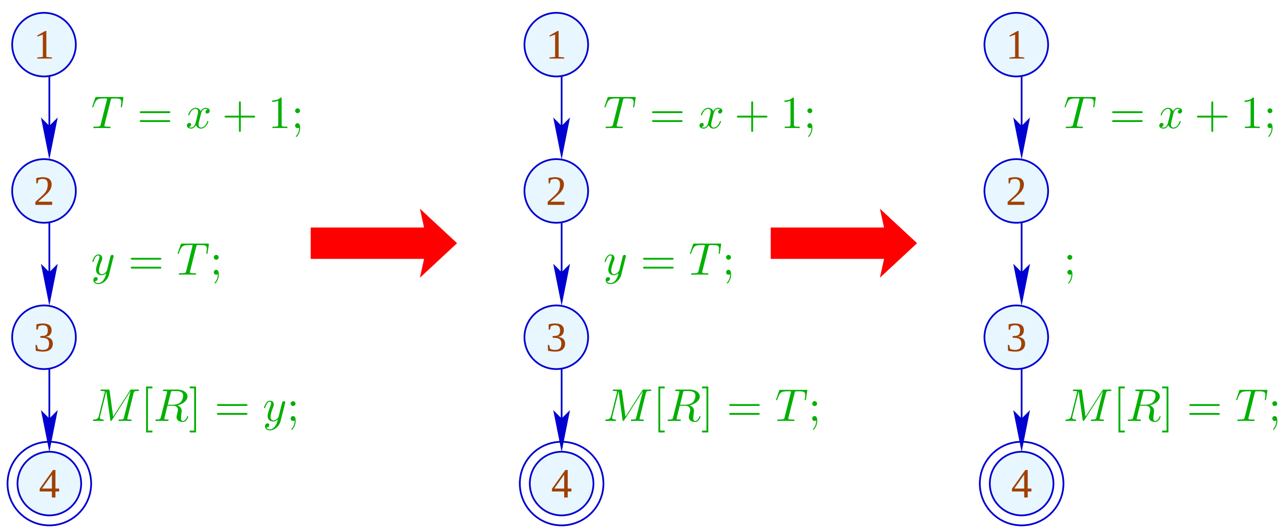

Advantage: Now, y has become dead :-))

Idea:

For each expression, we record the variable which currently contains its value :-)

We use: $\mathbb{V} = \mathit{Expr} \to 2^{\mathit{Vars}}$ ...