32 Predicates

A predicate q/k is defined through a sequence of clauses $rr \equiv r_1 \ldots r_f$.

The translation of q/k provides the translations of the individual clauses r_i.

In particular, we have for $f = 1$:

$$\text{code}_P\ rr \quad = \quad \text{code}_C\ r_1$$

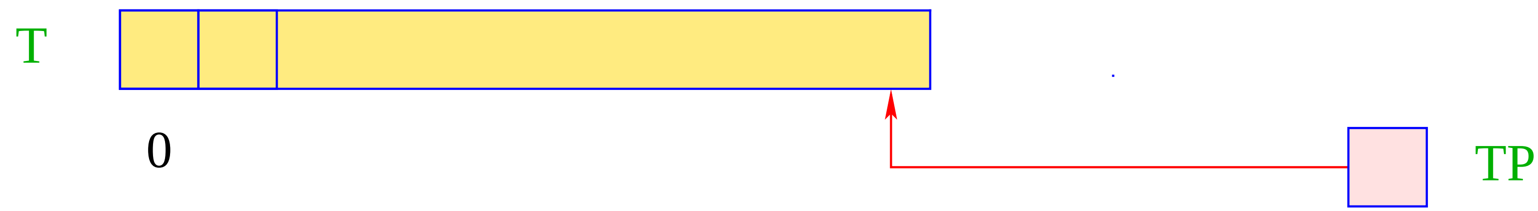
If q/k is defined through several clauses, the first alternative must be tried.

On failure, the next alternative must be tried

$\Longrightarrow$ backtracking :-)

32.1 Backtracking

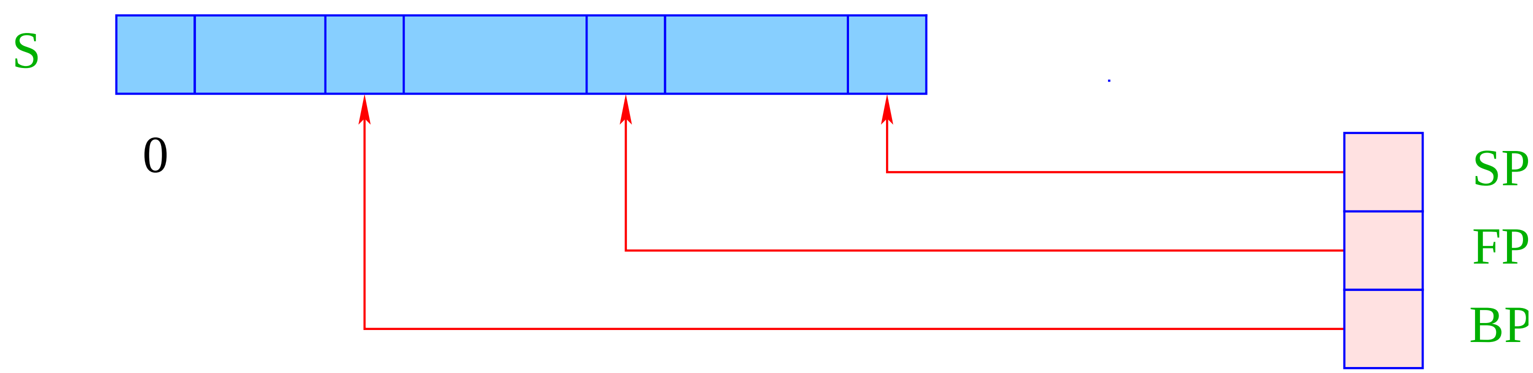
- Whenever unifcation fails, we call the run-time function `backtrack()`.
- The goal is to roll back the whole computation to the (dynamically :-) latest goal where another clause can be chosen $\Longrightarrow$ the last backtrack point.
- In order to undo intermediate variable bindings, we always have recorded new bindings with the run-time function `trail()`.
- The run-time function `trail()` stores variables in the data-structure trail:



TP $==$ Trail Pointer

points to the topmost occupied Trail cell

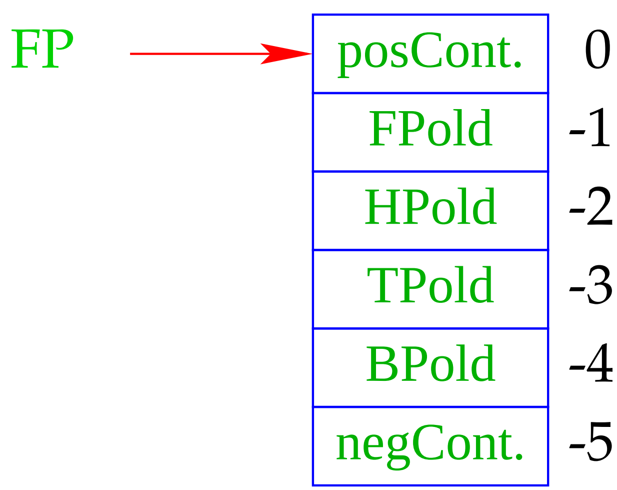
The current stack frame where backtracking should return to is pointed at by the extra register BP:



A backtrack point is stack frame to which program execution possibly returns.

- We need the code address for trying the next alternative (negative continuation address);
- We save the old values of the registers HP, TP and BP.
- Note: The new BP will receive the value of the current FP :-)

For this purpose, we use the corresponding four organizational cells:



For more comprehensible notation, we thus introduce the macros:

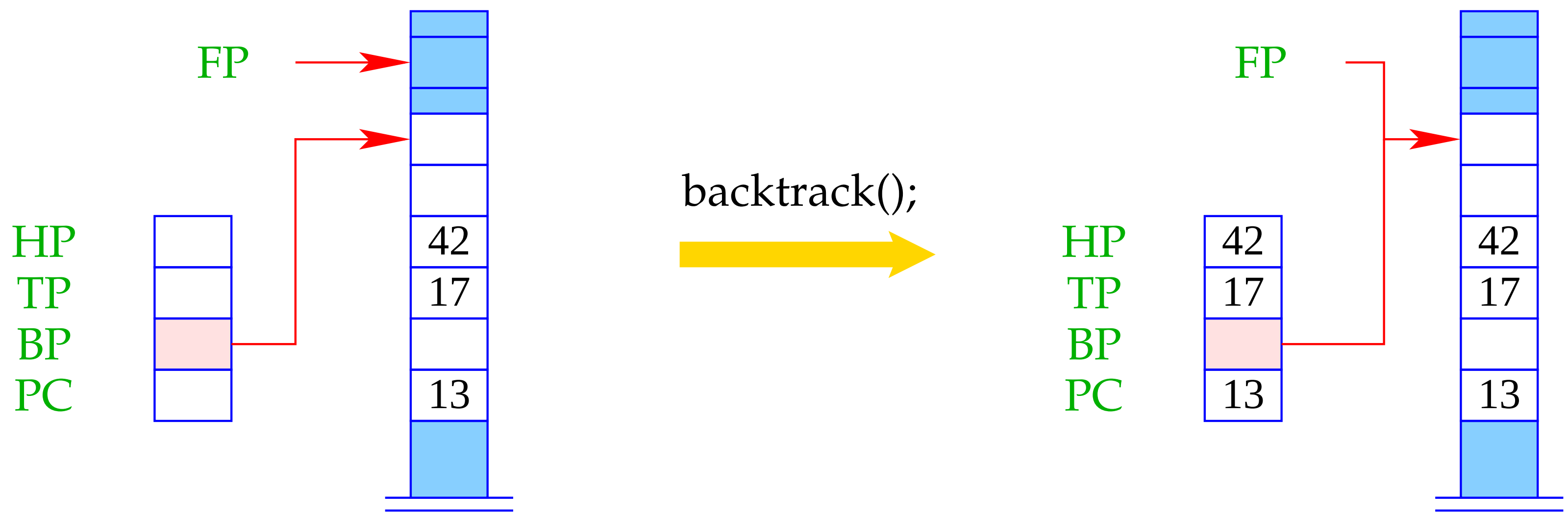
$$
\begin{array}{lcl}
\text{posCont} & \equiv & S[\text{FP}] \\
\text{FPold} & \equiv & S[\text{FP} - 1] \\
\text{HPold} & \equiv & S[\text{FP} - 2] \\
\text{TPold} & \equiv & S[\text{FP} - 3] \\
\text{BPold} & \equiv & S[\text{FP} - 4] \\
\text{negCont} & \equiv & S[\text{FP} - 5]
\end{array}
$$

for the corresponding addresses.

Remark:

Occurrence on the left	$\Longrightarrow$	saving the register
Occurrence on the right	$\Longrightarrow$	restoring the register

Calling the run-time function `void backtrack()` yields:




```
void backtrack() {
    FP = BP; HP = HPold;
    reset (TPold, TP);
    TP = TPold; PC = negCont;
}
```

where the run-time function `reset()` undoes the bindings of variables established since the backtrack point.

32.2 Resetting Variables

Idea:

- The variables which have been created since the last backtrack point can be removed together with their bindings by popping the heap !!! :-)
- This works fine if younger variables always point to older objects.
- Bindings of old variables to younger objects, though, must be reset manually :-(
- These are therefore recorded in the trail.

Functions `void trail(ref u)` and `void reset (ref y, ref x)` can thus be implemented as:

```
void trail (ref u) {
   if (u < S[BP-2]) {
      TP = TP+1;
      T[TP] = u;
   }
}
```

```
void reset (ref x, ref y) {
   for (ref u=y; x<u; u--)
      H[T[u]] = (R,T[u]);
}
```

Here, `S[BP-2]` represents the heap pointer when creating the last backtrack point.

32.3 Wrapping it Up

Assume that the predicate q/k is defined by the clauses $\quad r_1, \ldots, r_f \quad (f > 1)$.
We provide code for:

- setting up the backtrack point;
- successively trying the alternatives;
- deleting the backtrack point.

This means:

$$
\begin{array}{lllll}
\text{code}_P\ rr & = & q/k: & \text{setbtp} \\
& & & \text{try}\ A_1 \\
& & & \ldots \\
& & & \text{try}\ A_{f-1} \\
& & & \text{delbtp} \\
& & & \text{jump}\ A_f \\
& & A_1: & \text{code}_C\ r_1 \\
& & & \ldots \\
& & A_f: & \text{code}_C\ r_f
\end{array}
$$

Note:

- We delete the backtrack point before the last alternative :-)
- We jump to the last alternative — never to return to the present frame :-))

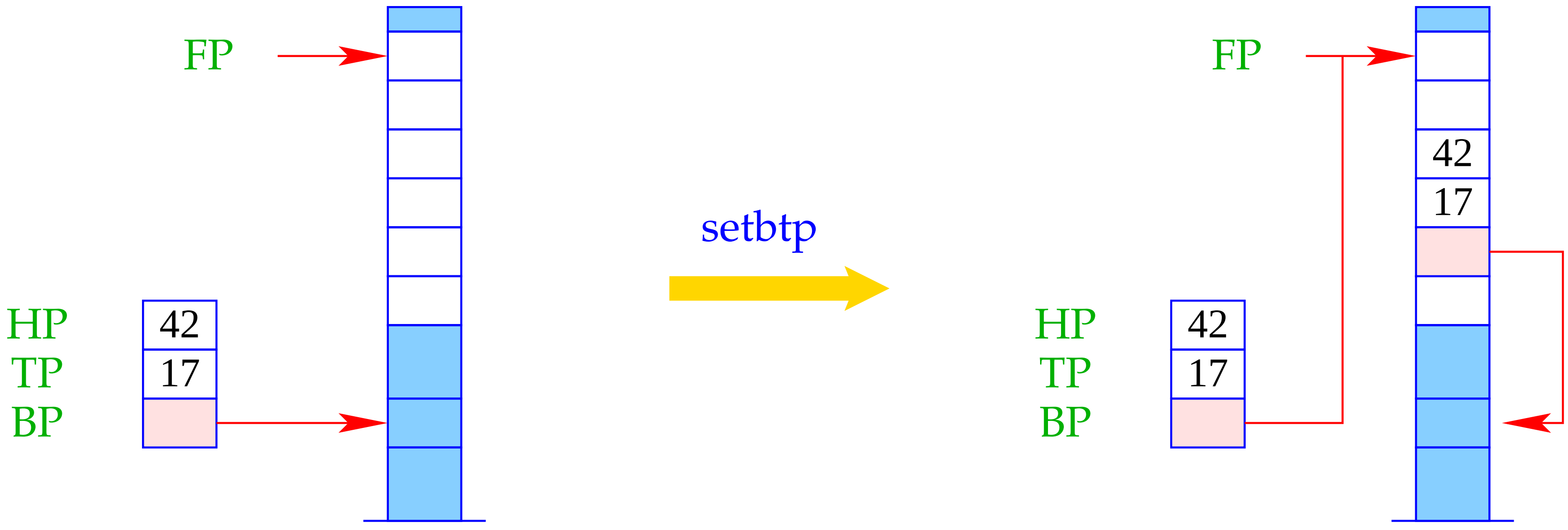
Example:

$$\begin{array}{lcl} \mathsf{s}(X) & \leftarrow & \mathsf{t}(\bar{X}) \\ \mathsf{s}(X) & \leftarrow & \bar{X} = a \end{array}$$

The translation of the predicate s yields:

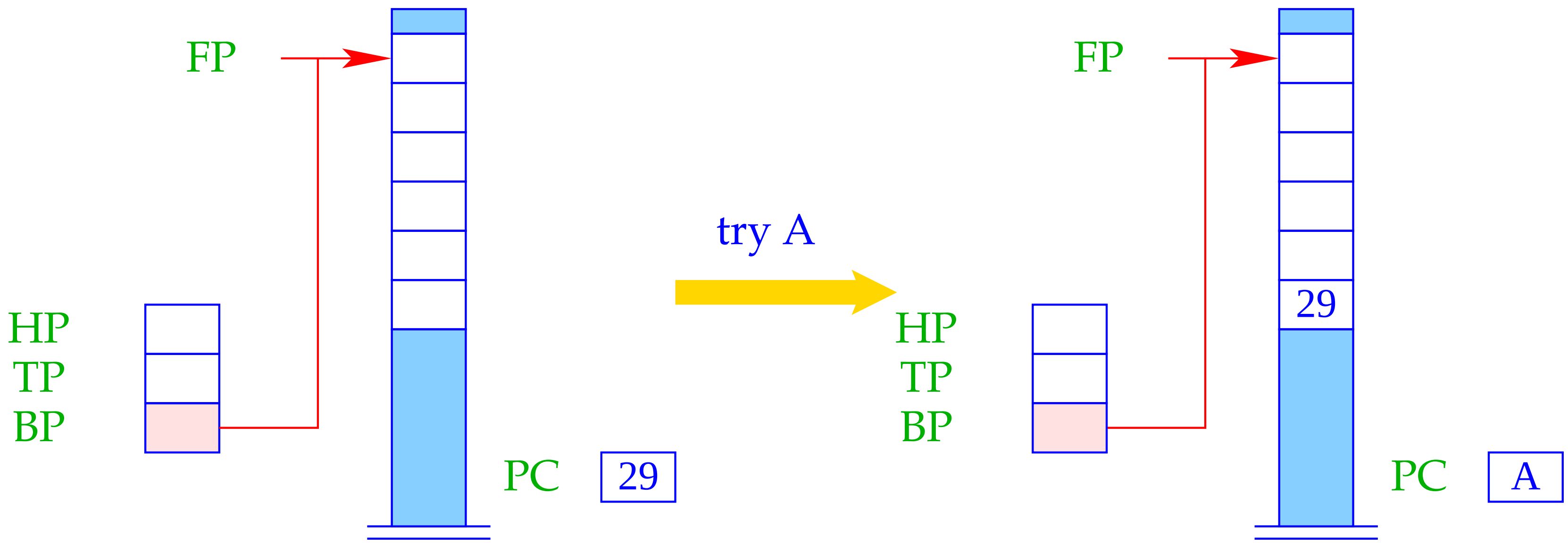
s/1:	setbtp	A:	pushenv 1	B:	pushenv 1
	try A		mark C		putref 1
	delbtp		putref 1		uatom a
	jump B		call t/1		popenv
		C:	popenv		

The instruction setbtp saves the registers HP, TP, BP:



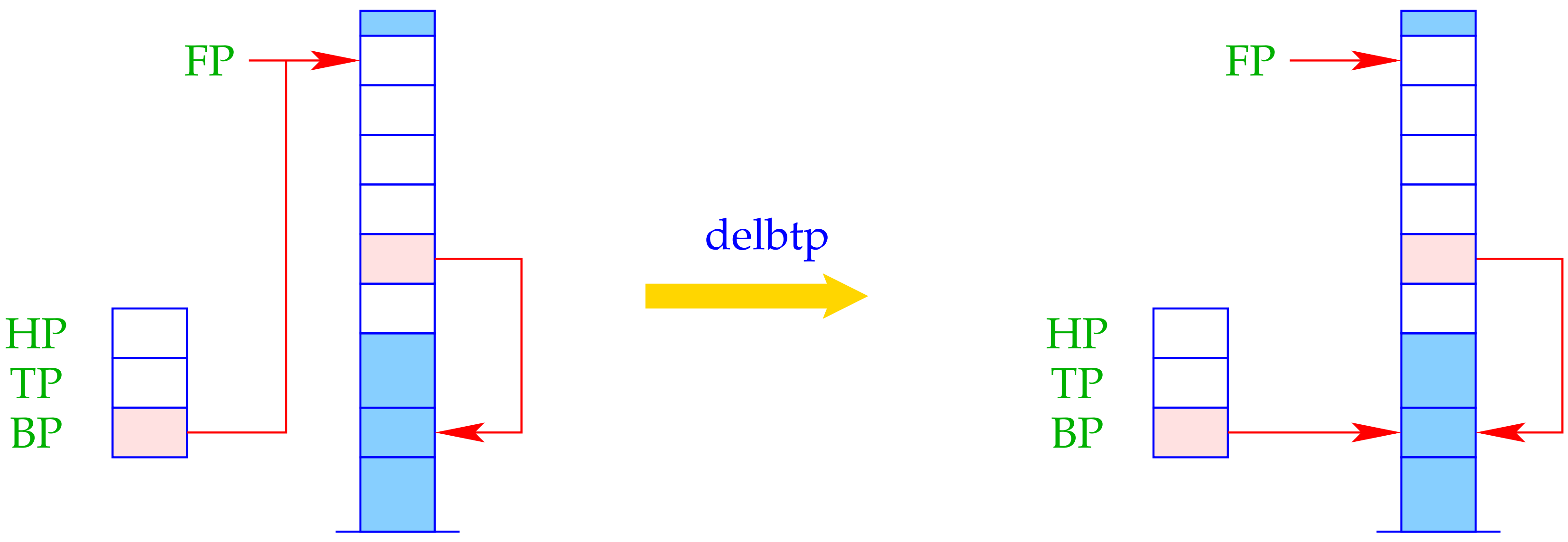
HPold = HP;
TPold = TP;
BPold = BP;
BP = FP;

The instruction try A tries the alternative at address A and updates the negative continuation address to the current PC:



negForts = PC;
PC = A;

The instruction delbtp restores the old backtrack pointer:



BP = BPold;

32.4 Popping of Stack Frames

Recall the translation scheme for clauses:

$$
\begin{aligned}
\text{code}_C\ r \quad = \quad & \text{pushenv m} \\
& \text{code}_G\ g_1\ \text{æ} \\
& \ldots \\
& \text{code}_G\ g_n\ \text{æ} \\
& \text{popenv}
\end{aligned}
$$

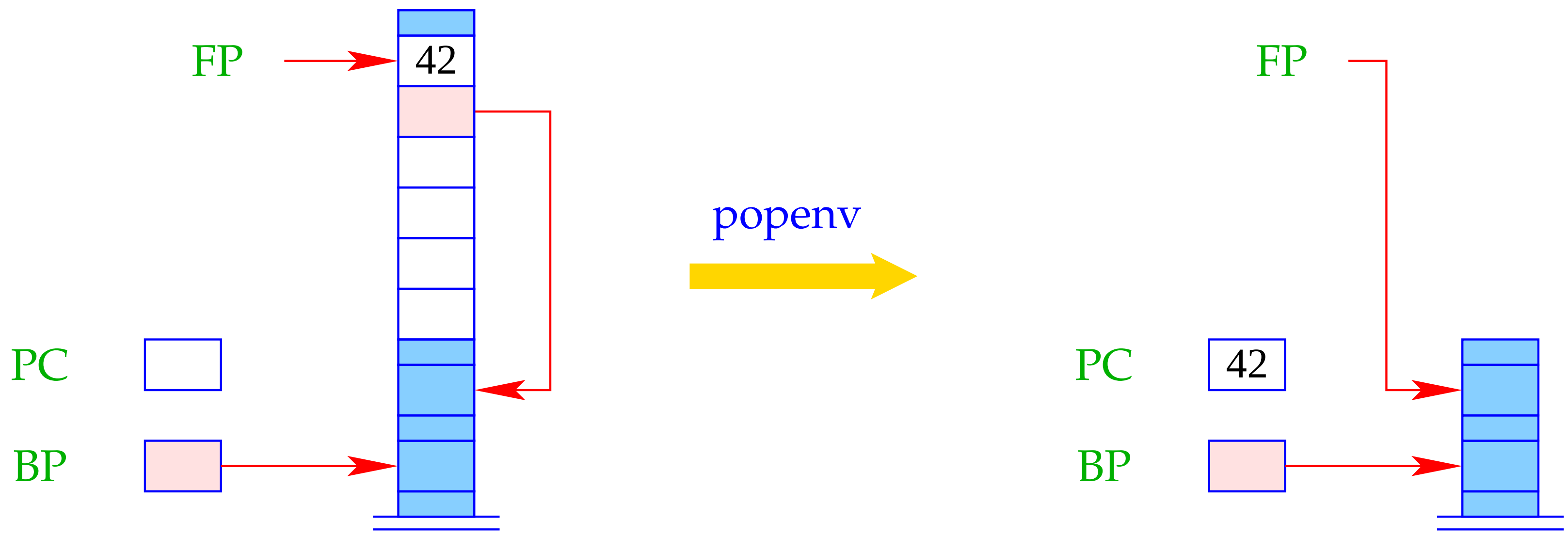
The present stack frame can be popped ...

- if the applied clause was the last (or only); and
- if all goals in the body are definitely finished.

$\Longrightarrow$ the backtrack point is older :-)

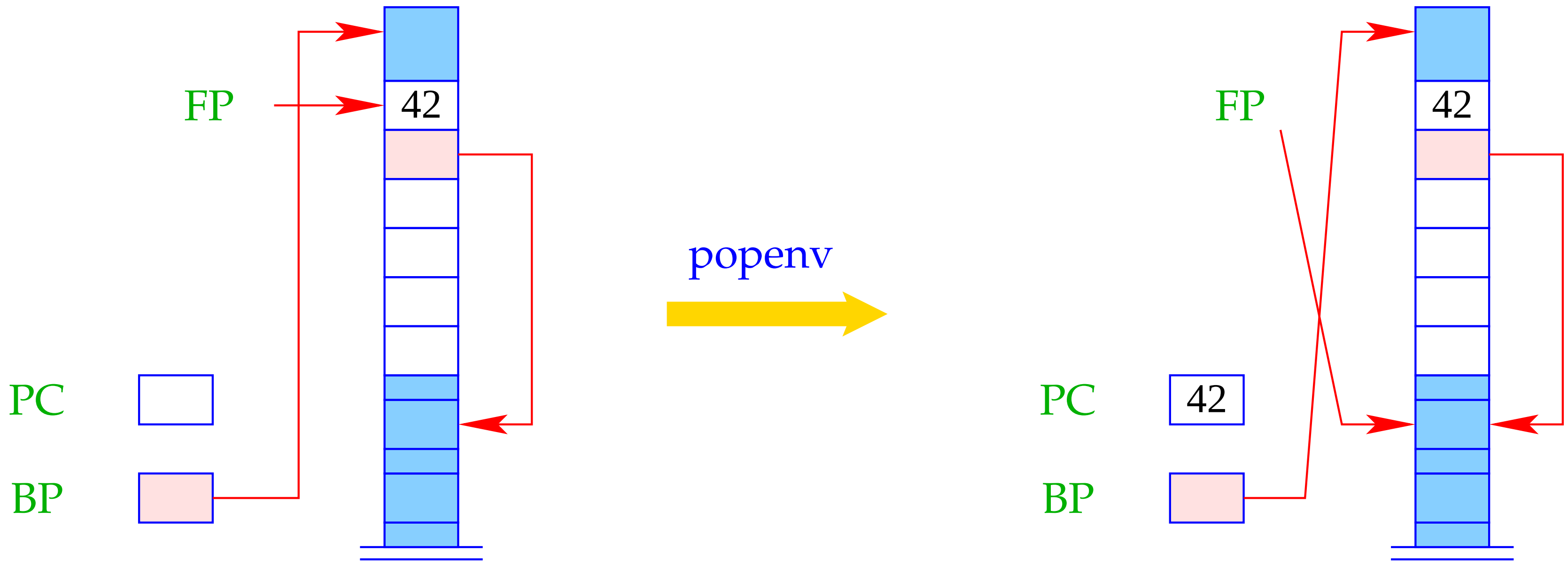
$\Longrightarrow$ FP > BP

The instruction popenv restores the registers FP and PC and possibly pops the stack frame:



if (FP > BP) SP = FP - 6;
PC = posCont;
FP = FPold;

Warning: popenv may fail to de-allocate the frame !!!



if (FP > BP) SP = FP - 6;
PC = posCont;
FP = FPold;

If popping the stack frame fails, new data are allocated on top of the stack. When returning to the frame, the locals still can be accessed through the FP :-))

33 Queries and Programs

The translation of a program: $p \equiv rr_1 \ldots rr_h ? g$
consists of:

- an instruction no for failure;
- code for evaluating the query g;
- code for the predicate definitions rr_i.

Preceding query evaluation:

$\Longrightarrow$ initialization of registers
$\Longrightarrow$ allocation of space for the globals

Succeeding query evaluation:

$\Longrightarrow$ returning the values of globals

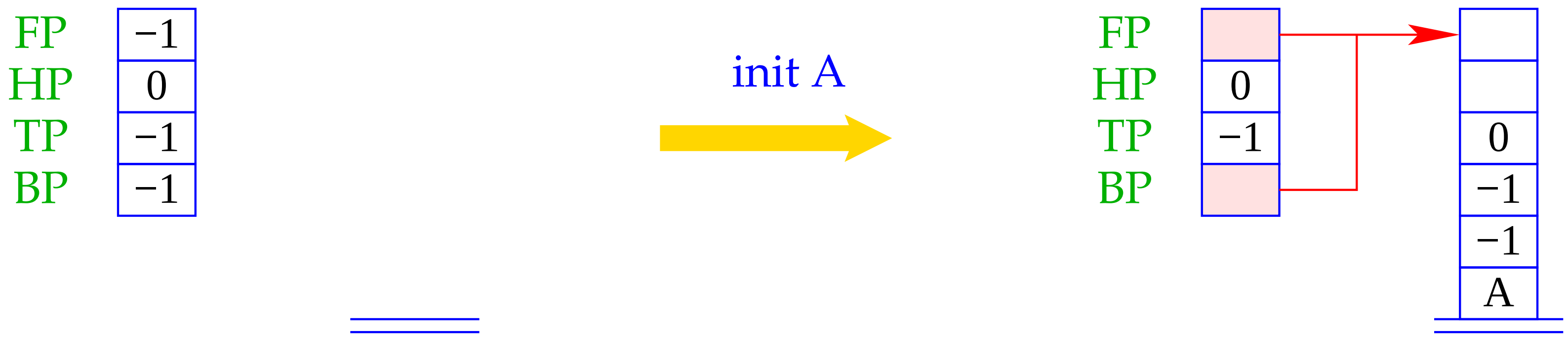
$$
\begin{array}{rrl}
\text{code } p & = & \text{init A} \\
& & \text{pushenv d} \\
& & \text{code}_G\ g\ æ \\
& & \text{halt d} \\
& \text{A:} & \text{no} \\
& & \text{code}_P\ rr_1 \\
& & \ldots \\
& & \text{code}_P\ rr_h
\end{array}
$$

where $free(g) = \{X_1, \ldots, X_d\}$ and æ is given by æ $X_i = i$.

The instruction halt d ...

- ... terminates the program execution;
- ... returns the bindings of the d globals;
- ... causes backtracking — if demanded by the user :-)

The instruction init A is defined by:



BP = FP = SP = 5;
S[0] = A;
S[1] = S[2] = -1;
S[3] = 0;
BP = FP;

At address "A" for a failing goal we have placed the instruction no for printing no to the standard output and halt :-)

The Final Example:

$$t(X) \leftarrow \bar{X} = b \qquad q(X) \leftarrow s(\bar{X}) \qquad s(X) \leftarrow \bar{X} = a$$

$$p \leftarrow q(X), t(\bar{X}) \qquad s(X) \leftarrow t(\bar{X}) \qquad ? \quad p$$

The translation yields:

	init N		popenv	q/1:	pushenv 1	E:	pushenv 1
	pushenv 0	p/0:	pushenv 1		mark D		mark G
	mark A		mark B		putref 1		putref 1
	call p/0		putvar 1		call s/1		call t/1
A:	halt 0		call q/1	D:	popenv	G:	popenv
N:	no	B:	mark C	s/1:	setbtp	F:	pushenv 1
t/1:	pushenv 1		putref 1		try E		putref 1
	putref 1		call t/1		delbtp		uatom a
	uatom b	C:	popenv		jump F		popenv

34 Last Call Optimization

Consider the app predicate from the beginnning:

$$\begin{array}{lll} \mathsf{app}(X, Y, Z) & \leftarrow & X = [\,], \; Y = Z \\ \mathsf{app}(X, Y, Z) & \leftarrow & X = [H|X'], \; Z = [H|Z'], \; \mathsf{app}(X', Y, Z') \end{array}$$

We observe:

- The recursive call occurs in the last goal of the clause.
- Such a goal is called last call.

$\Longrightarrow$ we try to evaluate it in the current stack frame !!!

$\Longrightarrow$ after (successful) completion, we will not return to the current caller !!!

Consider a clause r: $p(X_1, \ldots, X_k) \leftarrow g_1, \ldots, g_n$ with m locals where $g_n \equiv q(t_1, \ldots, t_h)$. The interplay between code_C and code_G:

$$
\begin{array}{rrl}
\text{code}_C\ r & = & \text{pushenv m} \\
 & & \text{code}_G\ g_1\ \text{æ} \\
 & & \ldots \\
 & & \text{code}_G\ g_{n-1}\ \text{æ} \\
 & & \text{mark B} \\
 & & \text{code}_A\ t_1\ \text{æ} \\
 & & \ldots \\
 & & \text{code}_A\ t_h\ \text{æ} \\
 & & \text{call q/h} \\
 & \text{B}: & \text{popenv}
\end{array}
$$

Replacement:	mark B	$\Longrightarrow$	lastmark
	call q/h; popenv	$\Longrightarrow$	lastcall q/h m

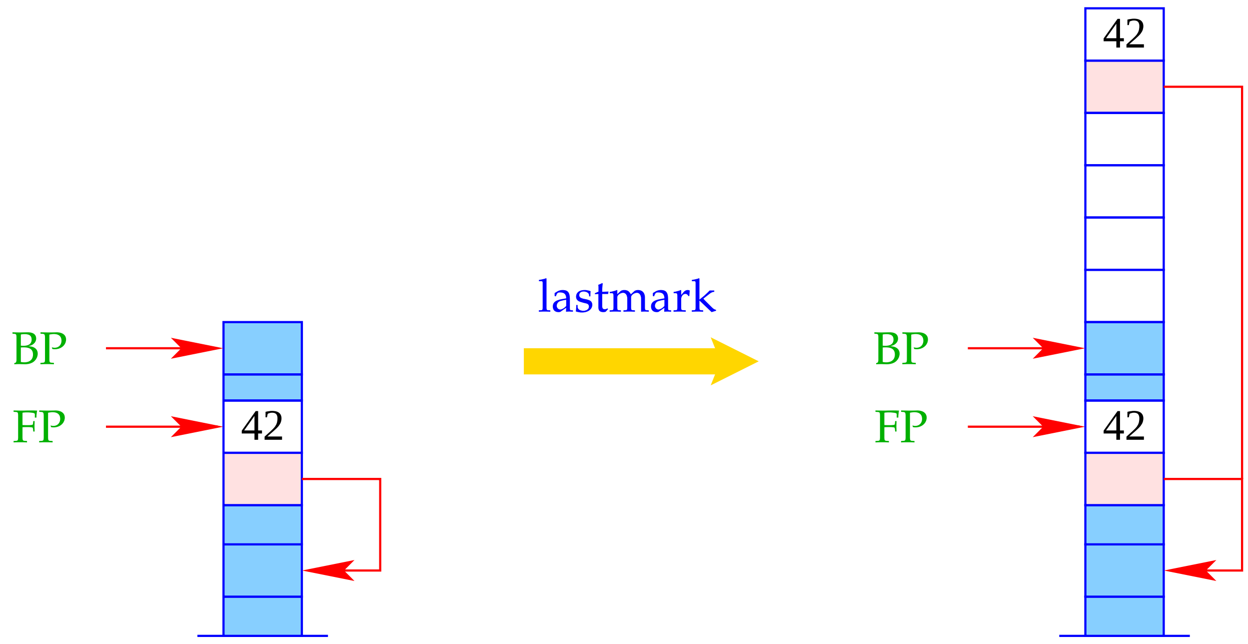
Consider a clause r: $\quad p(X_1, \ldots, X_k) \leftarrow g_1, \ldots, g_n$
with m locals where $\quad g_n \equiv q(t_1, \ldots, t_h)$. The interplay between code_C and code_G:

$$
\begin{array}{lcl}
\text{code}_C\ r & = & \text{pushenv m} \\
 & & \text{code}_G\ g_1\ \text{æ} \\
 & & \ldots \\
 & & \text{code}_G\ g_{n-1}\ \text{æ} \\
 & & \text{lastmark} \\
 & & \text{code}_A\ t_1\ \text{æ} \\
 & & \ldots \\
 & & \text{code}_A\ t_h\ \text{æ} \\
 & & \text{lastcall}\ \ \text{q/h}\ \ \text{m}
\end{array}
$$

Replacement:	mark B	$\Longrightarrow$	lastmark
	call q/h; popenv	$\Longrightarrow$	lastcall q/h m

If the current clause is not last or the $g_1, \ldots, g_{n-1}$ have created backtrack points, then FP $\leq$ BP :-)

Then lastmark creates a new frame but stores a reference to the predecessor:



if (FP $\leq$ BP) {
SP = SP + 6;
S[SP] = posCont; S[SP-1] = FPold;
}

If FP > BP then lastmark does nothing :-)

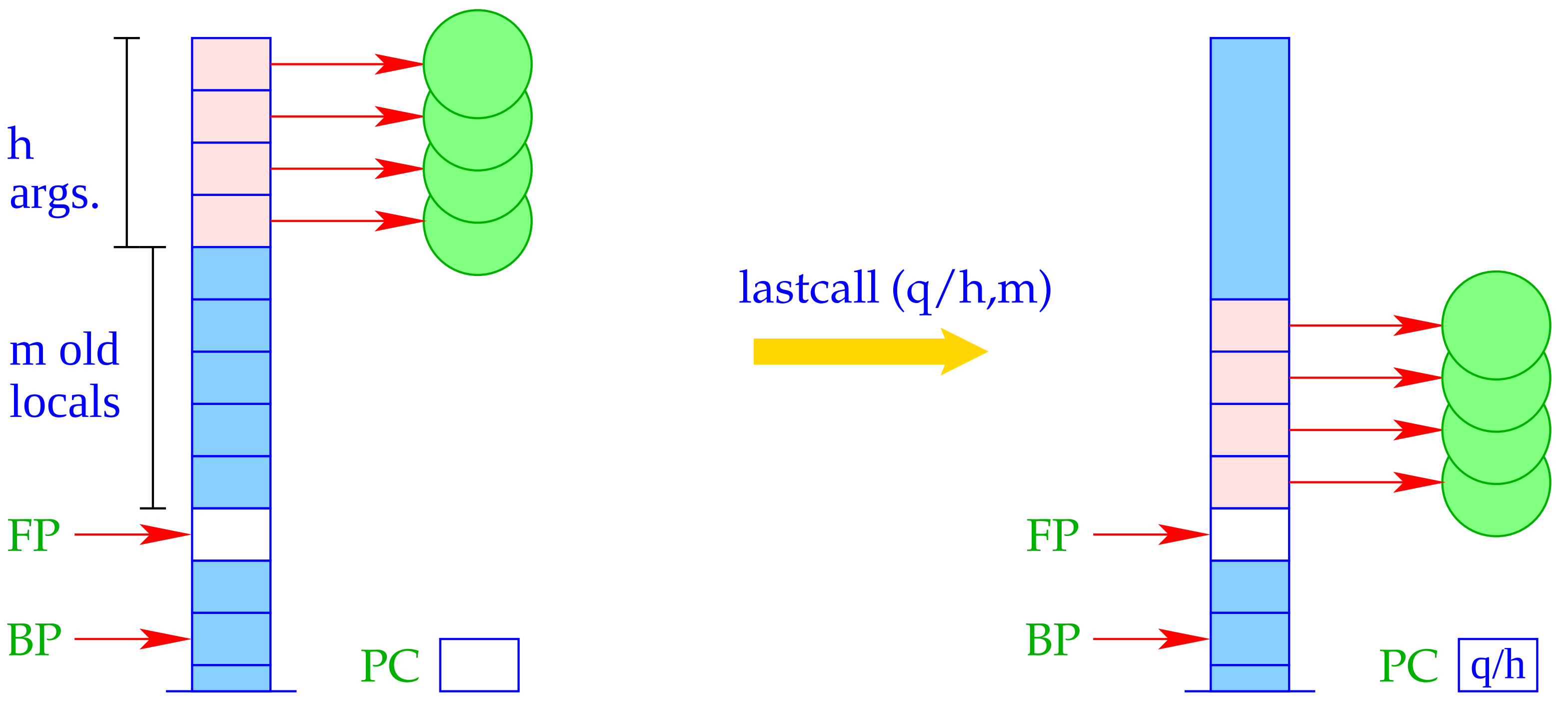
If $FP \leq BP$, then lastcall q/h m behaves like a normal call q/h.

Otherwise, the current stack frame is re-used. This means that:

- the cells S[FP+1], S[FP+2], . . ., S[FP+h] receive the new values and
- q/h can be jumped to :-)

```
lastcall q/h m   =   if (FP ≤ BP)  call q/h;
                     else {
                        move m h;
                        jump q/h;
                     }
```

The difference between the old and the new addresses of the parameters m just equals the number of the local variables of the current clause :-))



h
args.
m old
locals
FP
BP
PC
lastcall (q/h,m)
FP
BP
PC
q/h

Example:

Consider the clause:

$$\mathsf{a}(X, Y) \leftarrow \mathsf{f}(\bar{X}, X_1), \mathsf{a}(\bar{X}_1, \bar{Y})$$

The last-call optimization for code$_C$ r yields:

	mark A	A:	lastmark
pushenv 3	putref 1		putref 3
	putvar 3		putref 2
	call f/2		lastcall a/2 3

Example:

Consider the clause:

$$\mathsf{a}(X, Y) \leftarrow \mathsf{f}(\bar{X}, X_1), \mathsf{a}(\bar{X}_1, \bar{Y})$$

The last-call optimization for code$_C$ r yields:

	mark A	A:	lastmark
pushenv 3	putref 1		putref 3
	putvar 3		putref 2
	call f/2		lastcall a/2 3

Note:

If the clause is last and the last literal is the only one, we can skip lastmark and can replace lastcall q/h m with the sequence move m n; jump p/n :-))

Example:

Consider the last clause of the app predicate:

$$\mathsf{app}(X, Y, Z) \quad \leftarrow \quad \bar{X} = [H|X'],\ \bar{Z} = [\bar{H}|Z'],\ \mathsf{app}(\bar{X}', \bar{Y}, \bar{Z}')$$

Here, the last call is the only one :-) Consequently, we obtain:

```
A:  pushenv 6
    putref 1
    ustruct [|]/2 B
    son 1
    uvar 4
    son 2
    uvar 5
    up C
B:  putvar 4
    putvar 5
    putstruct [|]/2
    bind
C:  putref 3
    ustruct [|]/2 D
    son 1
    uref 4
    son 2
    uvar 6
    up E
D:  check 4
    putref 4
    putvar 6
    putstruct [|]/2
    bind
E:  putref 5
    putref 2
    putref 6
    move 6 3
    jump app/3
```

35 Trimming of Stack Frames

Idea:

- Order local variables according to their life times;
- Pop the dead variables — if possible :-}

35 Trimming of Stack Frames

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- Order local variables according to their life times;
- Pop the dead variables — if possible :-}

Example:

Consider the clause:

$$\mathsf{a}(X, Z) \leftarrow \mathsf{p}_1(\bar{X}, X_1), \mathsf{p}_2(\bar{X}_1, X_2), \mathsf{p}_3(\bar{X}_2, X_3), \mathsf{p}_4(\bar{X}_3, \bar{Z})$$

35 Trimming of Stack Frames

Idea:

- Order local variables according to their life times;
- Pop the dead variables — if possible :-}

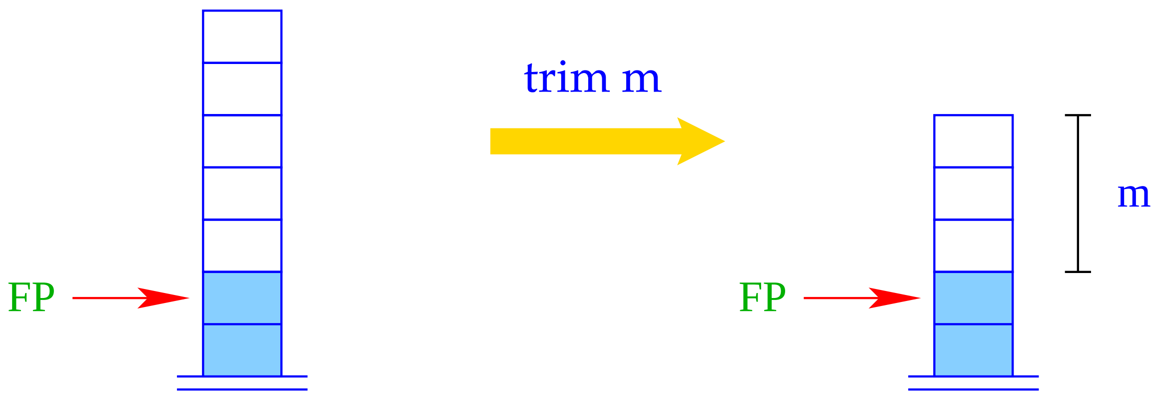
Example:

Consider the clause:

$$\mathsf{a}(X, Z) \leftarrow \mathsf{p}_1(\bar{X}, X_1), \mathsf{p}_2(\bar{X}_1, X_2), \mathsf{p}_3(\bar{X}_2, X_3), \mathsf{p}_4(\bar{X}_3, \bar{Z})$$

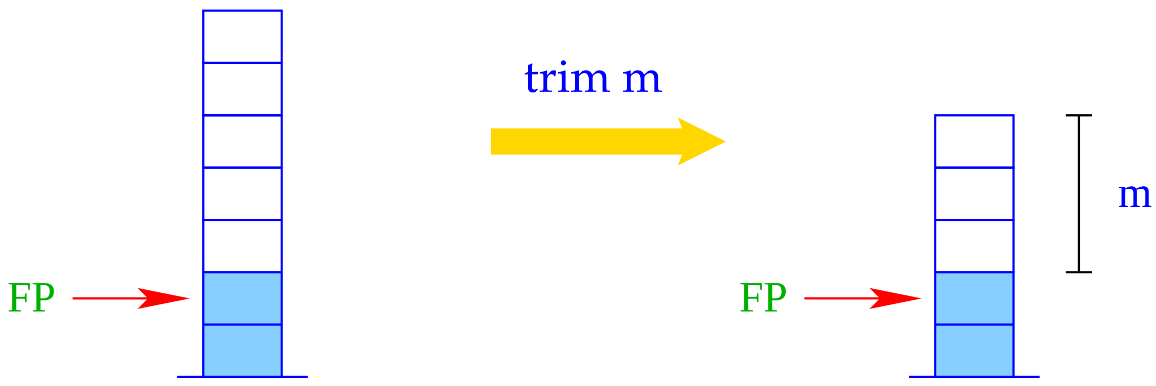
After the query $\mathsf{p}_2(\bar{X}_1, X_2)$, variable X_1 is dead.
After the query $\mathsf{p}_3(\bar{X}_2, X_3)$, variable X_2 is dead :-)

After every non-last goal with dead variables, we insert the instruction trim :



if (FP $\geq$ BP)
SP = FP + m;

After every non-last goal with dead variables, we insert the instruction trim :



if (FP $\geq$ BP)
SP = FP + m;

The dead locals can only be popped if no new backtrack point has been allocated :-)

Example (continued):

$$\mathsf{a}(X, Z) \leftarrow \mathsf{p}_1(\bar{X}, X_1), \mathsf{p}_2(\bar{X}_1, X_2), \mathsf{p}_3(\bar{X}_2, X_3), \mathsf{p}_4(\bar{X}_3, \bar{Z})$$

Ordering of the variables:

$$æ = \{X \mapsto 1, Z \mapsto 2, X_3 \mapsto 3, X_2 \mapsto 4, X_1 \mapsto 5\}$$

The resulting code:

pushenv 5	A:	mark B		mark C	lastmark
mark A		putref 5		putref 4	putref 3
putref 1		putvar 4		putvar 3	putref 2
putvar 5		call $\mathsf{p}_2/2$		call $\mathsf{p}_3/2$	lastcall $\mathsf{p}_4/2$ 3
call $\mathsf{p}_1/2$	B:	trim 4	C:	trim 3	

36 Clause Indexing

Observation:

Often, predicates are implemented by case distinction on the first argument.

$\Longrightarrow$ Inspecting the first argument, many alternatives can be excluded :-)

$\Longrightarrow$ Failure is earlier detected :-)

$\Longrightarrow$ Backtrack points are earlier removed. :-))

$\Longrightarrow$ Stack frames are earlier popped :-)))

Example: The app-predicate:

$$\begin{array}{lcl} \mathsf{app}(X, Y, Z) & \leftarrow & X = [\,],\ Y = Z \\ \mathsf{app}(X, Y, Z) & \leftarrow & X = [H|X'],\ Z = [H|Z'],\ \mathsf{app}(X', Y, Z') \end{array}$$

- If the root constructor is $[\,]$, only the first clause is applicable.
- If the root constructor is $[|]$, only the second clause is applicable.
- Every other root constructor should fail !!
- Only if the first argument equals an unbound variable, both alternatives must be tried ;-)

Idea:

- Introduce separate try chains for every possible constructor.
- Inspect the root node of the first argument.
- Depending on the result, perform an indexed jump to the appropriate try chain.

Assume that the predicate `p/k` is defined by the sequence rr of clauses $r_1 \ldots r_m$.

Let tchains rr denote the sequence of try chains as built up for the root constructors occurring in unifications $X_1 = t$.

Example:

Consider again the app-predicate, and assume that the code for the two clauses start at addresses A_1 and A_2, respectively.

Then we obtain the following four try chains:

VAR:	setbtp	// variables	NIL:	jump A_1	// atom [ ]
	try A_1				
	delbtp		CONS:	jump A_2	// constructor [|]
	jump A_2				
			ELSE:	fail	// default

Example:

Consider again the app-predicate, and assume that the code for the two clauses start at addresses A_1 and A_2, respectively.

Then we obtain the following four try chains:

VAR:	setbtp	// variables	NIL:	jump A_1	// atom [ ]
	try A_1				
	delbtp		CONS:	jump A_2	// constructor [|]
	jump A_2				
			ELSE:	fail	// default

The new instruction fail takes care of any constructor besides [ ] and [|] ...

$$\text{fail} \quad = \quad \texttt{backtrack()}$$

It directly triggers backtracking :-)