```
_fac:  enter q      loadc 1      A:  loadr -3        mul
       alloc 0      storer -3        loadr -3        storer -3
       loadr -3     return           loadc 1         return
       loadc 0      jump B           sub          B: return
       leq                           mark
       jumpz A                       loadc _fac
                                     call
                                     slide 0
```

where $\rho_{\text{fac}} : x \mapsto (L, -3)$ and $\mathsf{q} = 5$.

10 Translation of Whole Programs

Before program execution, we have:

$$\mathrm{SP} = -1 \qquad \mathrm{FP} = \mathrm{EP} = -1 \qquad \mathrm{PC} = 0 \qquad \mathrm{NP} = \mathrm{MAX}$$

Let $p \equiv V\_defs \quad F\_def_1 \ldots F\_def_n$, denote a program where F_def_i is the definition of a function f_i of which one is called main .

The code for the program p consists of:

- code for the function definitions F_def_i;
- code for the allocation of global variables;
- code for the call of int main();
- the instruction halt which returns control to the operating system together with the value at address 0.

Then we define:

$$
\begin{array}{lcll}
\text{code } p\ \emptyset & = & & \text{enter } (\mathrm{k}+4) \\
 & & & \text{alloc } (\mathrm{k}+1) \\
 & & & \text{mark} \\
 & & & \text{loadc \_main} \\
 & & & \text{call} \\
 & & & \text{slide k} \\
 & & & \text{halt} \\
 & & \_\mathrm{f}_1: & \text{code } F\_def_1\ \rho \\
 & & & \vdots \\
 & & \_\mathrm{f}_n: & \text{code } F\_def_n\ \rho
\end{array}
$$

where $\emptyset \ \widehat{=}$ empty address environment;

$\rho \ \widehat{=}$ global address environment;

$\mathrm{k} \ \widehat{=}$ size of the global variables

The Translation of Functional Programming Languages

11 The language PuF

We only regard a mini-language PuF ("Pure Functions").

We do not treat, as yet:

- Side effects;
- Data structures.

A program is an expression e of the form:

$$
\begin{array}{rcl}
e & ::= & b \mid x \mid (\Box_1\ e) \mid (e_1\ \Box_2\ e_2) \\
 & \mid & (\textbf{if}\ e_0\ \textbf{then}\ e_1\ \textbf{else}\ e_2) \\
 & \mid & (e'\ e_0 \ldots e_{k-1}) \\
 & \mid & (\textbf{fun}\ x_0 \ldots x_{k-1} \rightarrow e) \\
 & \mid & (\textbf{let}\ x_1 = e_1\ \textbf{in}\ e_0) \\
 & \mid & (\textbf{let rec}\ x_1 = e_1\ \textbf{and} \ldots \textbf{and}\ x_n = e_n\ \textbf{in}\ e_0)
\end{array}
$$

An expression is therefore

- a basic value, a variable, the application of an operator, or
- a function-application, a function-abstraction, or
- a **let**-expression, i.e. an expression with locally defined variables, or
- a **let-rec**-expression, i.e. an expression with simultaneously defined local variables.

For simplicity, we only allow `int` as basic type.

Example:

The following well-known function computes the factorial of a natural number:

$$\begin{array}{lcl} \textbf{let rec}\ \mathrm{fac} & = & \textbf{fun}\ x \to \textbf{if}\ x \leq 1\ \textbf{then}\ 1 \\ & & \textbf{else}\ x \cdot \mathrm{fac}\ (x-1) \\ \textbf{in}\ \mathrm{fac}\ 7 & & \end{array}$$

As usual, we only use the minimal amount of parentheses.

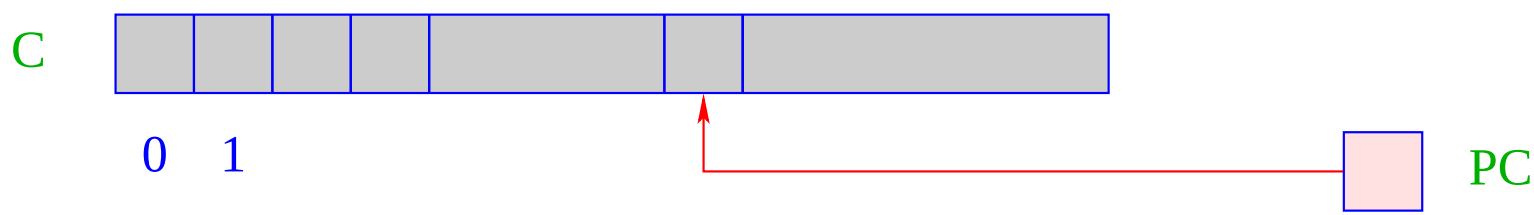
There are two Semantics:

CBV: Arguments are evaluated before they are passed to the function (as in SML);

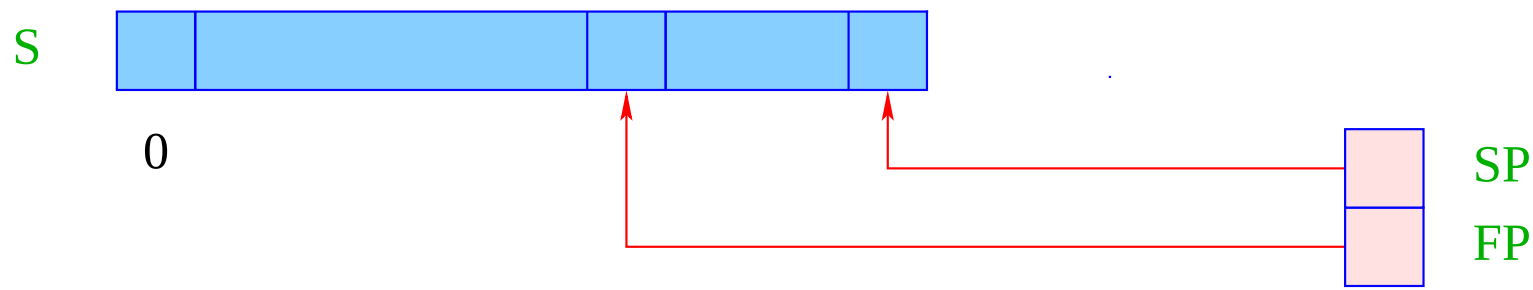
CBN: Arguments are passed unevaluated; they are only evaluated when their value is needed (as in Haskell).

12 Architecture of the MaMa:

We know already the following components:

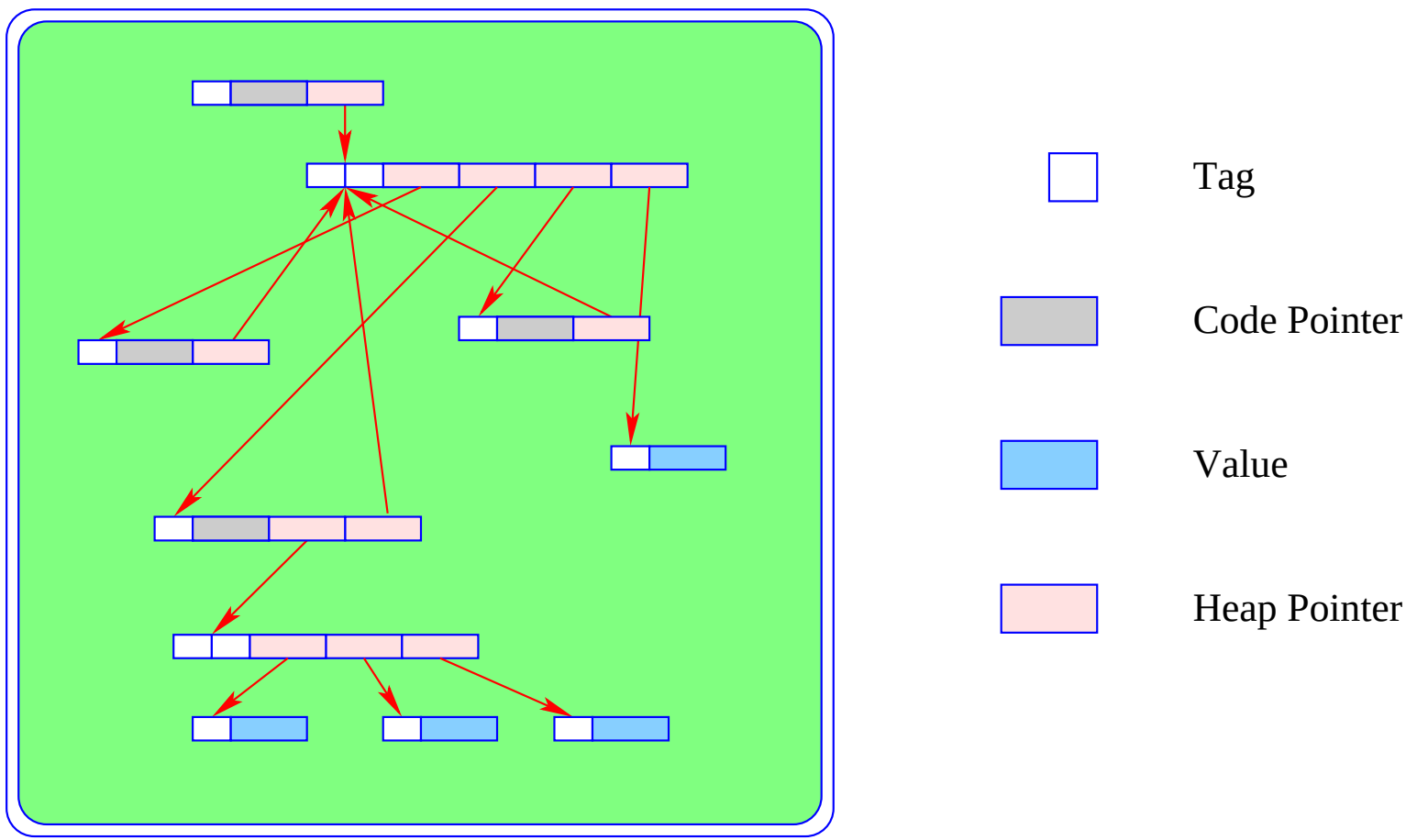


C	=	Code-store – contains the MaMa-program;
		each cell contains one instruction;
PC	=	Program Counter – points to the instruction to be executed next;

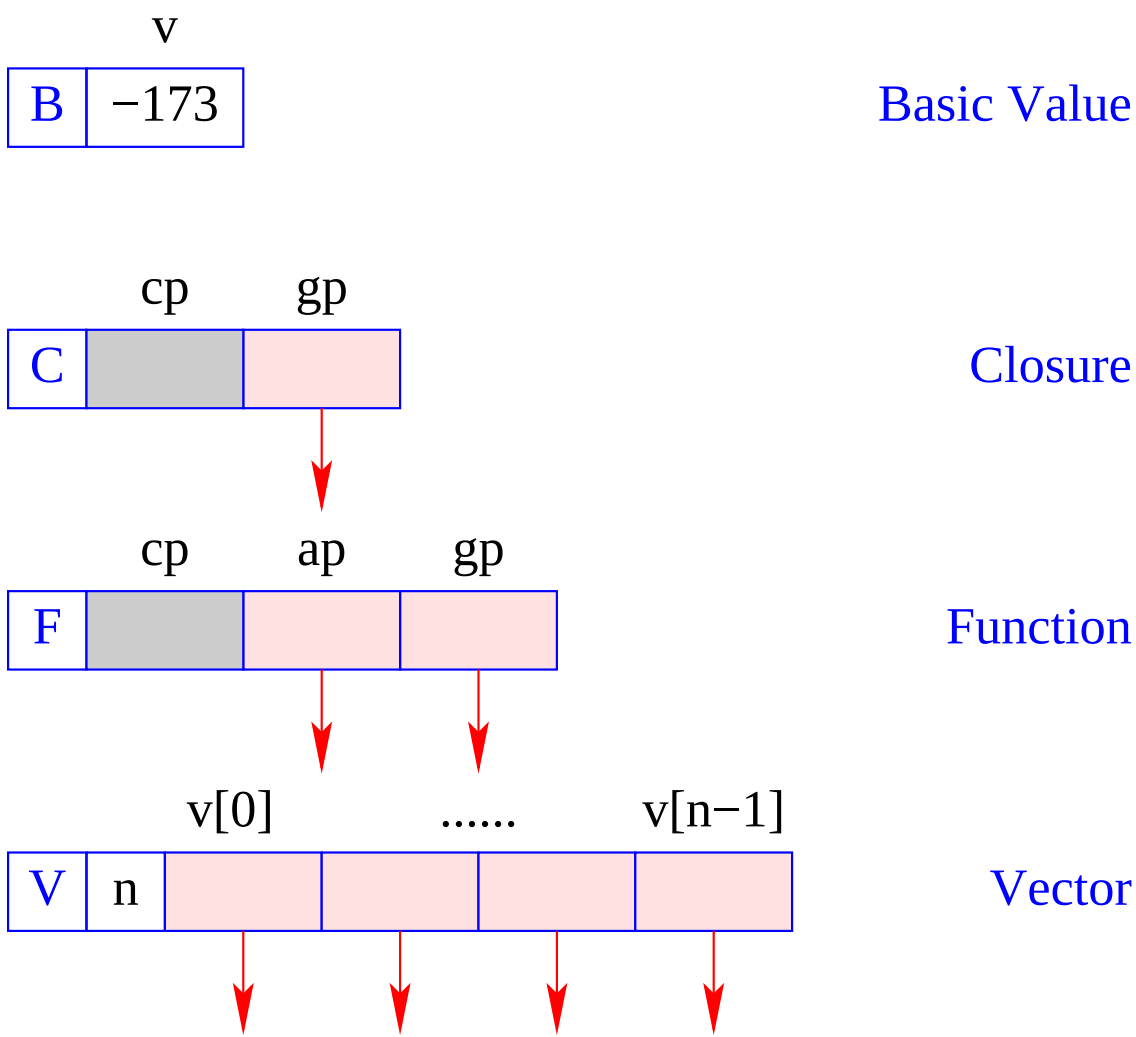


S	=	Runtime-Stack – each cell can hold a basic value or an address;
SP	=	Stack-Pointer – points to the topmost occupied cell;
		as in the CMa implicitely represented;
FP	=	Frame-Pointer – points to the actual stack frame.

We also need a heap H:



... it can be thought of as an abstract data type, being capable of holding data objects of the following form:



The instruction new $(tag, args)$ creates a corresponding object (B, C, F, V) in H and returns a reference to it.

We distinguish three different kinds of code for an expression e:

- $\text{code}_V\ e$ — (generates code that) computes the Value of e, stores it in the heap and returns a reference to it on top of the stack (the normal case);
- $\text{code}_B\ e$ — computes the value of e, and returns it on the top of the stack (only for Basic types);
- $\text{code}_C\ e$ — does not evaluate e, but stores a Closure of e in the heap and returns a reference to the closure on top of the stack.

We start with the code schemata for the first two kinds:

13 Simple expressions

Expressions consisting only of constants, operator applications, and conditionals are translated like expressions in imperative languages:

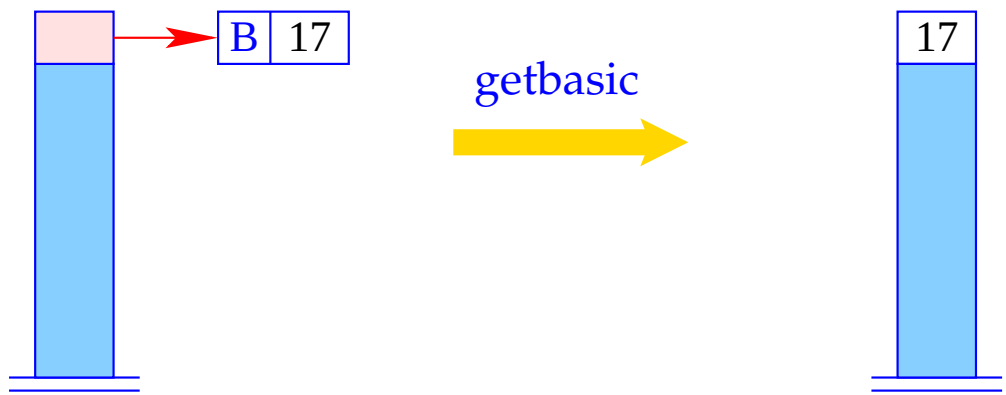
$$
\begin{array}{lcl}
\mathrm{code}_B\ b\ \rho\ \mathrm{sd} & = & \mathrm{loadc\ b} \\
\mathrm{code}_B\ (\Box_1\ e)\ \rho\ \mathrm{sd} & = & \mathrm{code}_B\ e\ \rho\ \mathrm{sd} \\
 & & \mathrm{op}_1 \\
\mathrm{code}_B\ (e_1\ \Box_2\ e_2)\ \rho\ \mathrm{sd} & = & \mathrm{code}_B\ e_1\ \rho\ \mathrm{sd} \\
 & & \mathrm{code}_B\ e_2\ \rho\ (\mathrm{sd}+1) \\
 & & \mathrm{op}_2
\end{array}
$$

$$
\begin{array}{lcll}
\text{code}_B\ (\textbf{if}\ e_0\ \textbf{then}\ e_1\ \textbf{else}\ e_2)\ \rho\ \text{sd} & = & & \text{code}_B\ e_0\ \rho\ \text{sd} \\
 & & & \text{jumpz A} \\
 & & & \text{code}_B\ e_1\ \rho\ \text{sd} \\
 & & & \text{jump B} \\
 & & \text{A:} & \text{code}_B\ e_2\ \rho\ \text{sd} \\
 & & \text{B:} & \ldots
\end{array}
$$

Note:

- ρ denotes the actual address environment, in which the expression is translated.
- The extra argument sd, the stack difference, *simulates* the movement of the SP when instruction execution modifies the stack. It is needed later to address variables.
- The instructions op_1 and op_2 implement the operators $\Box_1$ and $\Box_2$, in the same way as the the operators neg and add implement negation resp. addition in the CMa.
- For all other expressions, we first compute the value in the heap and then dereference the returned pointer:

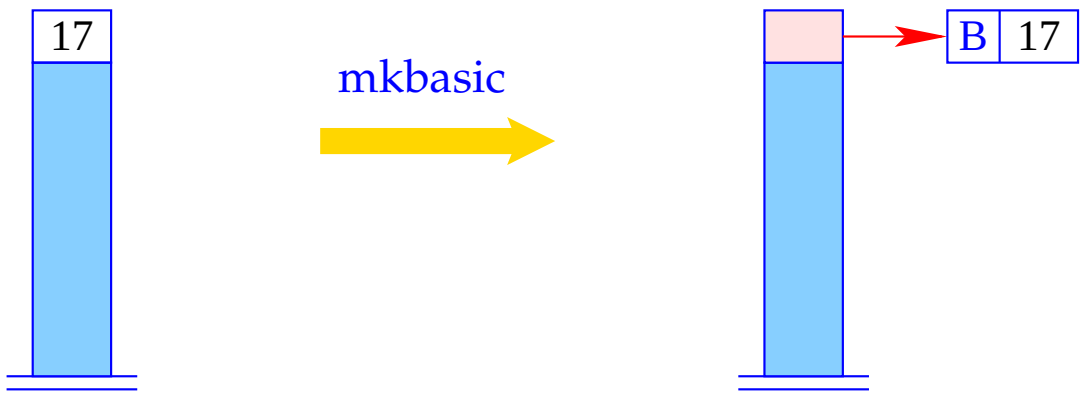
$$\begin{aligned} \mathrm{code}_B\ e\ \rho\ \mathrm{sd} \quad = \quad & \mathrm{code}_V\ e\ \rho\ \mathrm{sd} \\ & \mathrm{getbasic} \end{aligned}$$




```
if (H[S[SP]] != (B,_))
    Error "not basic!";
else
    S[SP] = H[S[SP]].v;
```

For code_V and simple expressions, we define analogously:

$$
\begin{array}{lcl}
\text{code}_V\ b\ \rho\ \text{sd} & = & \text{loadc b; mkbasic} \\
\text{code}_V\ (\Box_1\ e)\ \rho\ \text{sd} & = & \text{code}_B\ e\ \rho\ \text{sd} \\
 & & \text{op}_1\text{; mkbasic} \\
\text{code}_V\ (e_1\ \Box_2\ e_2)\ \rho\ \text{sd} & = & \text{code}_B\ e_1\ \rho\ \text{sd} \\
 & & \text{code}_B\ e_2\ \rho\ (\text{sd} + 1) \\
 & & \text{op}_2\text{; mkbasic} \\
\text{code}_V\ (\textbf{if}\ e_0\ \textbf{then}\ e_1\ \textbf{else}\ e_2)\ \rho\ \text{sd} & = & \text{code}_B\ e_0\ \rho\ \text{sd} \\
 & & \text{jumpz A} \\
 & & \text{code}_V\ e_1\ \rho\ \text{sd} \\
 & & \text{jump B} \\
 & & \text{A:}\quad \text{code}_V\ e_2\ \rho\ \text{sd} \\
 & & \text{B:}\quad \ldots
\end{array}
$$



S[SP] = new (B,S[SP]);

14 Accessing Variables

We must distinguish between local and global variables.

Example: Regard the function f :

$$
\begin{array}{lll}
\textbf{let} & c = 5 & \\
\textbf{in let} & f = \textbf{fun}\ a & \rightarrow \quad \textbf{let}\ b = a * a \\
 & & \quad\ \ \textbf{in}\ b + c \\
\textbf{in} & f\ c &
\end{array}
$$

The function f uses the global variable c and the local variables a (as formal parameter) and b (introduced by the inner **let**).

The binding of a global variable is determined, when the function is constructed (static scoping!), and later only looked up.