## Accessing Global Variables

- The bindings of global variables of an expression or a function are kept in a vector in the heap (Global Vector).
- They are addressed consecutively starting with 0 .
- When an F-object or a C-object are constructed, the Global Vector for the function or the expression is determined and a reference to it is stored in the gp-component of the object.
- During the evaluation of an expression, the (new) register GP (Global Pointer) points to the actual Global Vector.
- In constrast, local variables should be administered on the stack ...
$\Longrightarrow$ General form of the address environment:

$$
\rho: \text { Vars } \rightarrow\{L, G\} \times \mathbb{Z}
$$

## Accessing Local Variables

Local variables are administered on the stack, in stack frames.
Let $e \equiv e^{\prime} e_{0} \ldots e_{m-1}$ be the application of a function $e^{\prime}$ to arguments $e_{0}, \ldots, e_{m-1}$.

Warning:

The arity of $e^{\prime}$ does not need to be $\left.m \quad:-\right)$

- $f$ may therefore receive less than $n$ arguments (under supply);
- $f$ may also receive more than $n$ arguments, if $t$ is a functional type (over supply).


## Possible stack organisations:



+ Addressing of the arguments can be done relative to FP
- The local variables of $e^{\prime}$ cannot be addressed relative to FP.
- If $e^{\prime}$ is an $n$-ary function with $n<m$, i.e., we have an over-supplied function application, the remaining $m-n$ arguments will have to be shifted.
- If $e^{\prime}$ evaluates to a function, which has already been partially applied to the parameters $a_{0}, \ldots, a_{k-1}$, these have to be sneaked in underneath $e_{0}$ :



## Alternative:



+ The further arguments $a_{0}, \ldots, a_{k-1}$ and the local variables can be allocated above the arguments.

- Addressing of arguments and local variables relative to FP is no more possible. (Remember: $m$ is unknown when the function definition is translated.)


## Way out:

- We address both, arguments and local variables, relative to the stack pointer SP !!!
- However, the stack pointer changes during program execution...

- The differerence between the current value of SP and its value $\mathrm{sp}_{0}$ at the entry of the function body is called the stack distance, sd.
- Fortunately, this stack distance can be determined at compile time for each program point, by simulating the movement of the SP.
- The formal parameters $x_{0}, x_{1}, x_{2}, \ldots$ successively receive the non-positive relative addresses $0,-1,-2, \ldots$, i.e., $\quad \rho x_{i}=(L,-i)$.
- The absolute address of the $i$-th formal parameter consequently is

$$
\mathrm{sp}_{0}-i=(\mathrm{SP}-\mathrm{sd})-i
$$

- The local let-variables $y_{1}, y_{2}, y_{3}, \ldots$ will be successively pushed onto the stack:

- The $y_{i}$ have positive relative addresses $1,2,3, \ldots$, that is: $\quad \rho y_{i}=(L, i)$.
- The absolute address of $y_{i}$ is then $\quad \mathrm{sp}_{0}+i=(\mathrm{SP}-\mathrm{sd})+i$

With CBN, we generate for the access to a variable:

$$
\begin{aligned}
\operatorname{code}_{V} x \rho \mathrm{sd}= & \underset{\text { eval }}{\operatorname{getvar} x \rho \mathrm{sd}}
\end{aligned}
$$

The instruction eval checks, whether the value has already been computed or whether its evaluation has to yet to be done $(\Longrightarrow$ will be treated later :-) With CBV, we can just delete eval from the above code schema.
The (compile-time) macro getvar is defined by:

$$
\begin{aligned}
& \text { getvar } x \rho \mathrm{sd}= \text { let }(t, i)=\rho x \text { in } \\
& \text { match } t \text { with } \\
& L \rightarrow \text { pushloc }(\mathrm{sd}-i) \\
& \mid G \rightarrow \text { pushglob } \mathrm{i} \\
& \text { end }
\end{aligned}
$$

The access to local variables:


## Correctness argument:

Let sp and sd be the values of the stack pointer resp. stack distance before the execution of the instruction. The value of the local variable with address $i$ is loaded from $S[a]$ with

$$
a=\mathrm{sp}-(\mathrm{sd}-i)=(\mathrm{sp}-\mathrm{sd})+i=\mathrm{sp}_{0}+i
$$

... exactly as it should be :-)

The access to global variables is much simpler:


Example:
Regard $\quad e \equiv(b+c)$ for $\rho=\{b \mapsto(L, 1), c \mapsto(G, 0)\}$ and $\quad \mathrm{sd}=1$.
With CBN, we obtain:

$$
\begin{aligned}
& \operatorname{code}_{V} e \rho 1=\text { getvar } b \rho 1=1 \text { pushloc } 0 \\
& \text { eval } 2 \text { eval } \\
& \text { getbasic } 2 \text { getbasic } \\
& \text { getvar c } \rho 2 \quad 2 \text { pushglob } 0 \\
& \text { eval } 3 \text { eval } \\
& \text { getbasic } 3 \text { getbasic } \\
& \text { add } 3 \text { add } \\
& \text { mkbasic } 2 \text { mkbasic }
\end{aligned}
$$

## 15 let-Expressions

As a warm-up let us first consider the treatment of local variables :-)
Let $\quad e \equiv$ let $y_{1}=e_{1}$ in $\ldots$ let $e_{n}$ in $e_{0} \quad$ be a nested let-expression.
The translation of $e$ must deliver an instruction sequence that

- allocates local variables $y_{1}, \ldots, y_{n}$;
- in the case of

CBV: evaluates $e_{1}, \ldots, e_{n}$ and binds the $y_{i}$ to their values;
CBN: constructs closures for the $e_{1}, \ldots, e_{n}$ and binds the $y_{i}$ to them;

- evaluates the expression $e_{0}$ and returns its value.

Here, we consider the non-recursive case only, i.e. where $y_{j}$ only depends on $y_{1}, \ldots, y_{j-1}$. We obtain for CBN:

$$
\begin{aligned}
\operatorname{code}_{V} e \rho \mathrm{sd}= & \operatorname{code}_{C} e_{1} \rho \mathrm{sd} \\
& \operatorname{code}_{C} e_{2} \rho_{1}(\mathrm{sd}+1) \\
& \ldots \\
& \operatorname{code}_{C} e_{n} \rho_{n-1}(\mathrm{sd}+n-1) \\
& \operatorname{code}_{V} e_{0} \rho_{n}(\mathrm{sd}+n)
\end{aligned}
$$

$$
\text { slide } \mathrm{n} \quad / / \text { deallocates local variables }
$$

where

$$
\rho_{j}=\rho \oplus\left\{y_{i} \mapsto(L, \mathrm{sd}+i) \mid i=1, \ldots, j\right\}
$$

In the case of CBV, we use code ${ }_{V}$ for the expressions $e_{1}, \ldots, e_{n}$.

## Warning!

All the $e_{i}$ must be associated with the same binding for the global variables!

## Example:

Consider the expression

$$
e \equiv \text { let } a=19 \text { in let } b=a * a \text { in } a+b
$$

for $\rho=\emptyset$ and $s d=0$. We obtain (for CBV):

| 0 | loadc 19 | 3 | getbasic | 3 | pushloc 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | mkbasic | 3 | mul | 4 | getbasic |
| 1 | pushloc 0 | 2 | mkbasic | 4 | add |
| 2 | getbasic | 2 | pushloc 1 | 3 | mkbasic |
| 2 | pushloc 1 | 3 | getbasic | 3 | slide 2 |

The instruction slide k deallocates again the space for the locals:


## 16 Function Definitions

The definition of a function $f$ requires code that allocates a functional value for $f$ in the heap. This happens in the following steps:

- Creation of a Global Vector with the binding of the free variables;
- Creation of an (initially empty) argument vector;
- Creation of an F-Object, containing references to theses vectors and the start address of the code for the body;

Separately, code for the body has to be generated.
Thus:

$$
\begin{aligned}
& \operatorname{code}_{V}\left(\text { fun } x_{0} \ldots x_{k-1} \rightarrow e\right) \rho \text { sd }=\quad \text { getvar } z_{0} \rho \mathrm{sd} \\
& \text { getvar } z_{1} \rho(\mathrm{sd}+1) \\
& \text { getvar } z_{g-1} \rho(\mathrm{sd}+g-1) \\
& \text { mkvec } g \\
& \text { mkfunval A } \\
& \text { jump B } \\
& \text { A: } \operatorname{targ} k \\
& \text { code }_{V} \text { e } \rho^{\prime} 0 \\
& \text { return } \mathrm{k} \\
& \text { B : ... }
\end{aligned}
$$

where $\quad\left\{z_{0}, \ldots, z_{g-1}\right\}=$ free $\left(\right.$ fun $\left.x_{0} \ldots x_{k-1} \rightarrow e\right)$
and $\quad \rho^{\prime}=\left\{x_{i} \mapsto(L,-i) \mid i=0, \ldots, k-1\right\} \cup\left\{z_{j} \mapsto(G, j) \mid j=0, \ldots, g-1\right\}$

mkvec g


$$
\begin{aligned}
& \mathrm{h}=\text { new }(\mathrm{V}, \mathrm{n}) ; \\
& \mathrm{SP}=\mathrm{SP}-\mathrm{g}+1 ; \\
& \text { for }(\mathrm{i}=0 ; \mathrm{i}<\mathrm{g} ; \mathrm{i}++) \\
& \quad \mathrm{h} \rightarrow \mathrm{v}[\mathrm{i}]=\mathrm{S}[\mathrm{SP}+\mathrm{i}] ; \\
& \mathrm{S}[\mathrm{SP}]=\mathrm{h} ;
\end{aligned}
$$



Example:
Regard $\quad f \equiv$ fun $b \rightarrow a+b \quad$ for $\quad \rho=\{a \mapsto(L, 1)\}$ and $\quad \mathrm{sd}=1$. code $_{V} f \rho 1$ produces:

| 1 | pushloc 0 | 0 | pushglob 0 | 2 | getbasic |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | mkvec 1 | 1 | eval | 2 | add |
| 2 | mkfunval A | 1 | getbasic | 1 |  |
| mkbasic |  |  |  |  |  |
| 2 | jump B | 1 | pushloc 1 | 1 |  |
| 0 | A $: \operatorname{targ} 1$ | 2 | eval | 2 | B: |

The secrets around $\operatorname{targ} \mathrm{k}$ and return k will be revealed later :-)

## 17 Function Application

Function applications correspond to function calls in C. The necessary actions for the evaluation of $\quad e^{\prime} e_{0} \ldots e_{m-1}$ are:

- Allocation of a stack frame;
- Transfer of the actual parameters, i.e. with:

CBV: Evaluation of the actual parameters;
CBN: Allocation of closures for the actual parameters;

- Evaluation of the expression $e^{\prime}$ to an F-object;
- Application of the function.

Thus for CBN:

$$
\begin{array}{rlrl}
\operatorname{code}_{V}\left(e^{\prime} e_{0} \ldots e_{m-1}\right) \rho \mathrm{sd}= & \operatorname{mark} \mathrm{A} & / / \text { Allocation of the fra } \\
& \operatorname{code}_{C} e_{m-1} \rho(\mathrm{sd}+3) \\
& \operatorname{code}_{C} e_{m-2} \rho(\mathrm{sd}+4) \\
& \ldots \\
& \operatorname{code}_{C} e_{0} \rho(\mathrm{sd}+m+2) \\
& \operatorname{code}_{V} e^{\prime} \rho(\mathrm{sd}+m+3) \quad / / \text { Evaluation of } e^{\prime} \\
& \text { apply } & / / \text { corresponds to call } \\
A: & \ldots &
\end{array}
$$

To implement CBV, we use $\operatorname{code}_{V}$ instead of $\operatorname{code}_{C}$ for the arguments $e_{i}$.

Example: For $(f 42), \rho=\{f \mapsto(L, 2)\}$ and $s d=2$, we obtain with CBV:

| 2 | mark A | 6 | mkbasic | 7 |  | apply |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | loadc 42 | 6 | pushloc 4 | 3 | A: | $\ldots$ |

## A Slightly Larger Example:

$$
\text { let } a=17 \text { in let } f=\text { fun } b \rightarrow a+b \text { in } f 42
$$

For CBV and $s d=0$ we obtain:

| 0 | loadc 17 | 2 |  | jump B | 2 |  | getbasic | 5 |  | loadc 42 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | mkbasic | 0 | A: | $\operatorname{targ} 1$ | 2 |  | add | 5 |  | mkbasic |
| 1 | pushloc 0 | 0 |  | pushglob 0 | 1 |  | mkbasic | 6 |  | pushloc 4 |
| 2 | mkvec 1 | 1 |  | getbasic | 1 |  | return 1 | 7 |  | apply |
| 2 | mkfunval A | 1 |  |  | pushloc 1 | 2 | B: | mark C | 3 | C: |
| mlide 2 |  |  |  |  |  |  |  |  |  |  |

For the implementation of the new instruction, we must fix the organization of a stack frame:


Different from the CMa, the instruction mark A already saves the return address:


The instruction apply unpacks the F-object, a reference to which (hopefully) resides on top of the stack, and continues execution at the address given there:


$$
\begin{array}{ll}
\mathrm{h}=\mathrm{S}[\mathrm{SP}] ; & \mathrm{GP}=\mathrm{h} \rightarrow \mathrm{gp} ; \mathrm{PC}=\mathrm{h} \rightarrow \mathrm{cp} ; \\
\text { if }(\mathrm{H}[\mathrm{~h}]!=(\mathrm{F},-,-)) & \text { for }(\mathrm{i}=0 ; \mathrm{i}<\mathrm{h} \rightarrow \mathrm{ap} \rightarrow \mathrm{n} ; \mathrm{i}++) \\
\quad \text { Error "no fun"; } & \mathrm{S}[\mathrm{SP}+\mathrm{i}]=\mathrm{h} \rightarrow \mathrm{ap} \rightarrow \mathrm{v}[\mathrm{i}] ; \\
\text { else }\{ & \mathrm{SP}=\mathrm{SP}+\mathrm{h} \rightarrow \mathrm{ap} \rightarrow \mathrm{n}-1 ;
\end{array}
$$

## Warning:

- The last element of the argument vector is the last to be put onto the stack. This must be the first argument reference.
- This should be kept in mind, when we treat the packing of arguments of an under-supplied function application into an F-object!!!

