Call-Graph:

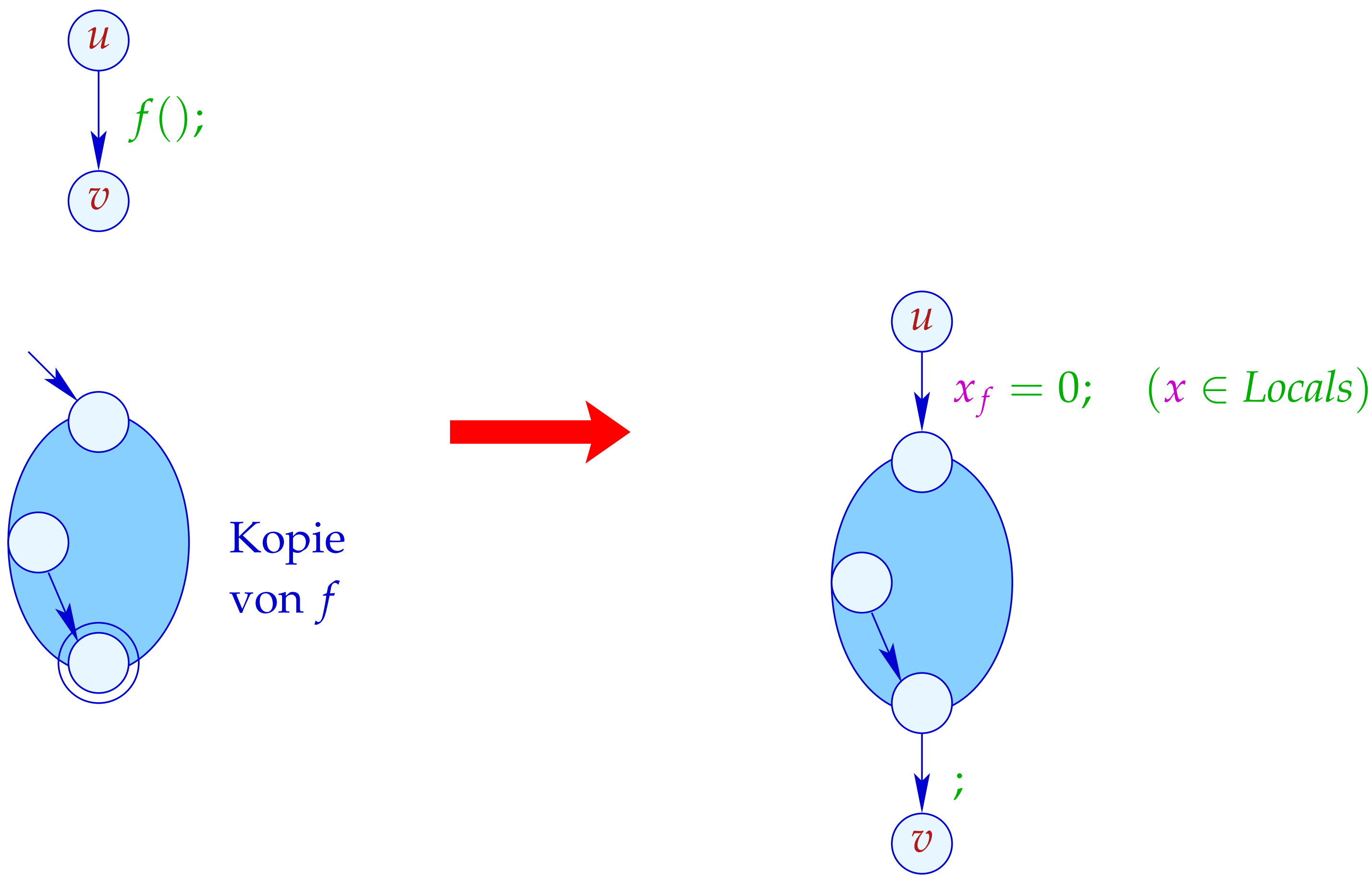
- The nodes are the procedures.
- An edge connexts g with h, whenever the body of g contains a call of h.

Strategies for Inlining:

- Just copy nur leaf-procedures, i.e., procedures without further calls :-)
- Copy all non-recursive procedures!

... here, we consider just leaf-procedures ;-)

Transformation 9:



Note:

- The Nop-edge can be eliminated if the *stop*-node of f has no out-going edges ...
- The x_f are the copies of the locals of the procedure f.
- According to our semantics of procedure calls, these must be initialized with 0 :-)

2. Idea: Elimination of Tail Recursion

$$
\begin{array}{l}
f\,()\ \{\quad \mathsf{int}\ \ b; \\
\qquad \mathsf{if}\ \ (a_2 \leq 1)\ \ \{\ \ \mathsf{ret} = a_1;\ \mathsf{goto}\ \_exit;\ \ \} \\
\qquad b = a_1 \cdot a_2; \\
\qquad a_2 = a_2 - 1; \\
\qquad a_1 = b; \\
\qquad f\,(); \\
\_exit\ \ : \\
\}
\end{array}
$$

After the procedure call, nothing in the body remains to be done.

$\Longrightarrow$ We may directly jump to the beginning :-)

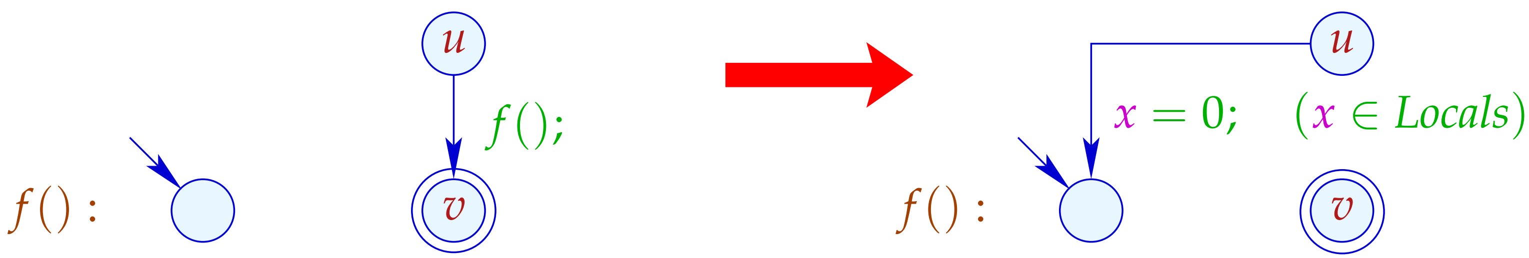
... after having reset the locals to 0.

... this yields in the Example:

$$
\begin{array}{ll}
f\,()\ \{ & \mathsf{int}\ \ b; \\
\_f: & \mathsf{if}\ \ (a_2 \leq 1)\ \ \{\ \ \mathsf{ret} = a_1;\ \mathsf{goto}\ \_exit;\ \ \} \\
 & b = a_1 \cdot a_2; \\
 & a_2 = a_2 - 1; \\
 & a_1 = b; \\
 & b = 0;\ \ \mathsf{goto}\ \ \_f; \\
\_exit: & \\
\} &
\end{array}
$$

// It works, since we have ruled out references to variables!

Transformation 11:



Warning:

- → This optimization is crucial for programming languages without iteration constructs !!!
- → Duplication of code is not necessary :-)
- → No variable renaming is necessary :-)
- → The optimization may also be profitable for non-recursive tail calls :-)
- → The corresponding code may contain jumps from the body of one procedure into the body of another ???

Background 4: Interprocedural Analysis

So far, we can analyze each procedure separately.

→ The costs are moderate :-)

→ The methods also work in presence of separate compilation :-)

→ At procedure calls, we must assume the worst case :-(

→ Constant propagation only works for local constants :-((

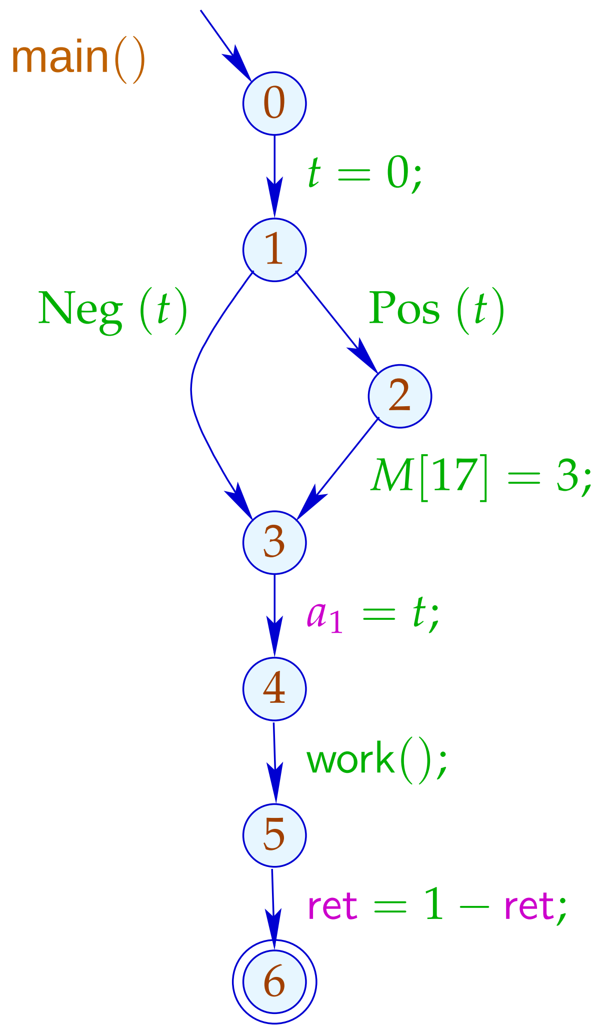
Question:

How can recursive programs be analyzed ???

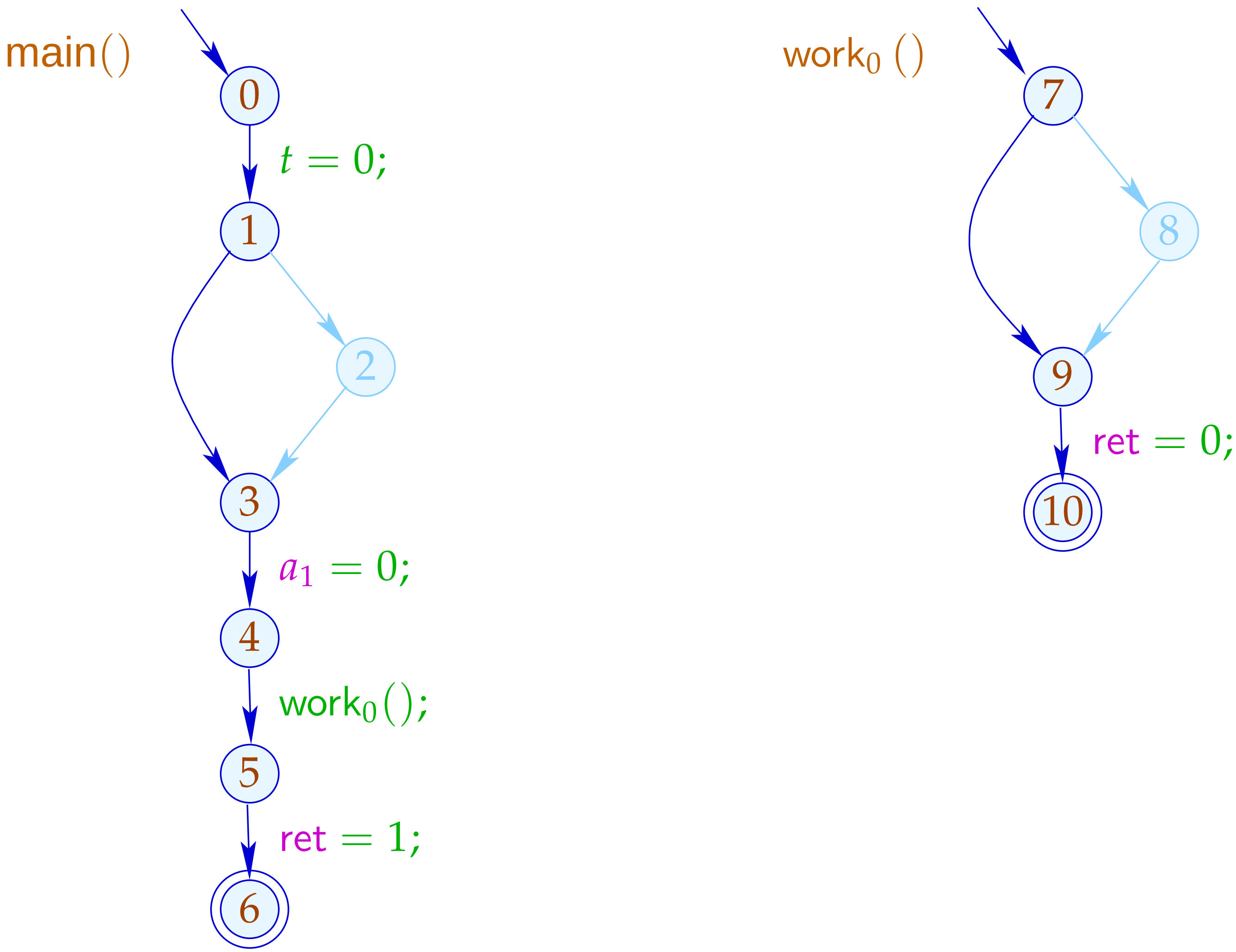
Example: Constant Propagation

$$
\begin{array}{ll}
\text{main}() \;\; \{ \; \textbf{int} \; t; & \text{work}() \;\; \{ \\
\qquad t = 0; & \qquad \text{if} \;\; (a_1) \;\; \text{work}(); \\
\qquad \text{if} \;\; (t) \;\; M[17] = 3; & \qquad \text{ret} = a_1; \\
\qquad a_1 = t; & \} \\
\qquad \text{work}\,(); & \\
\qquad \text{ret} = 1 - \text{ret}; & \\
\} &
\end{array}
$$

Example: Constant Propagation



Example: Constant Propagation



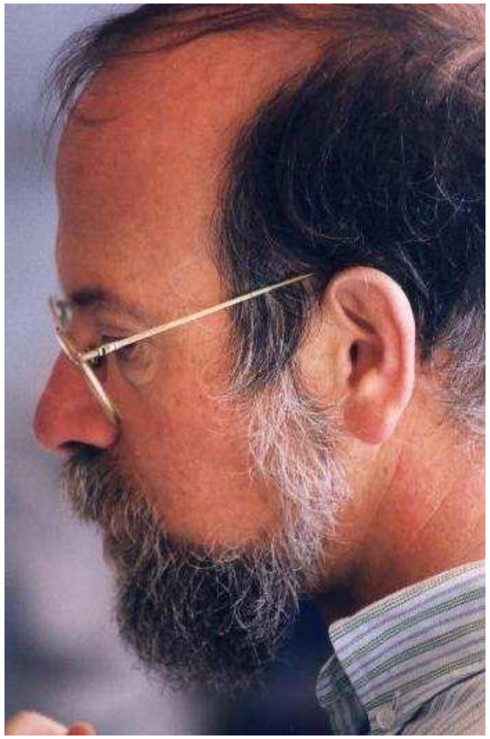
(1) Functional Approach:

Let $\mathbb{D}$ denote a complete lattice of (abstract) states.

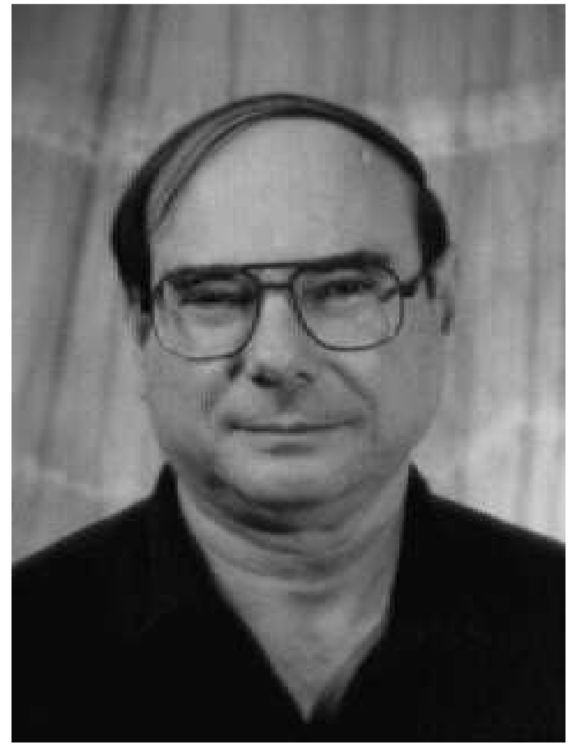
Idea:

Represent the effect of $f()$ by a function:

$$[\![f]\!]^\sharp : \mathbb{D} \to \mathbb{D}$$



Micha Sharir, Tel Aviv University



Amir Pnueli, Weizmann Institute

In order to determine the effect of a call edge $k = (u, f\,();, v)$ we require abstract functions:

$$\begin{array}{lll} \mathsf{enter}^\sharp & : & \mathbb{D} \longrightarrow \mathbb{D} \\ \mathsf{combine}^\sharp & : & \mathbb{D}^2 \longrightarrow \mathbb{D} \end{array}$$

Then we define:

$$[\![k]\!]^\sharp\, D \quad = \quad \mathsf{combine}^\sharp\,(D, [\![f]\!]^\sharp\,(\mathsf{enter}^\sharp\, D))$$

... for Constant Propagation:

$$\mathbb{D} = (Vars \to \mathbb{Z}^\top)_\perp$$

$$\mathsf{enter}^\sharp\; D = \begin{cases} \perp & \text{if} \quad D = \perp \\ D|_{Globals} \oplus \{x \mapsto 0 \mid x \in Locals\} & \text{otherwise} \end{cases}$$

$$\mathsf{combine}^\sharp\; (D_1, D_2) = \begin{cases} \perp & \text{if} \quad D_1 = \perp \vee D_2 = \perp \\ D_1|_{Locals} \oplus D_2|_{Globals} & \text{otherwise} \end{cases}$$

The effects $[\![f]\!]^\sharp$ then can be determined by a system of constraints over the complete lattice $\mathbb{D} \to \mathbb{D}$:

$$
\begin{array}{lllll}
[\![v]\!]^\sharp & \sqsupseteq & \mathsf{Id} & & v \quad \text{Eintrittspunkt} \\
[\![v]\!]^\sharp & \sqsupseteq & [\![k]\!]^\sharp \circ [\![u]\!]^\sharp & & k = (u, \_, v) \quad \text{edge} \\
[\![f]\!]^\sharp & \sqsupseteq & [\![stop_f]\!]^\sharp & & stop_f \quad \text{end point of} \quad f
\end{array}
$$

$[\![v]\!]^\sharp \; : \; \mathbb{D} \to \mathbb{D}$ describes the effect of all prefixes of computation forests w of a procedure which lead from the entry point to v :-)

Problems:

- How can we represent functions $f : \mathbb{D} \to \mathbb{D}$???
- If $\#\mathbb{D} = \infty$, then $\mathbb{D} \to \mathbb{D}$ has infinite strictly increasing chains :-(

Simplification: Copy-Constants

$\longrightarrow$ Conditions are interpreted as $;$:-)

$\longrightarrow$ Only assignments $x = e;$ with $e \in \mathit{Vars} \cup \mathbb{Z}$ are treated exactly :-)

Observation:

→ The effects of assignments are:

$$[\![x = e;]\!]^\sharp\, D \;=\; \begin{cases} D \oplus \{x \mapsto c\} & \text{if} \quad e = c \in \mathbb{Z} \\ D \oplus \{x \mapsto (D\; y)\} & \text{if} \quad e = y \in \mathit{Vars} \\ D \oplus \{x \mapsto \top\} & \text{otherwise} \end{cases}$$

→ Let $\mathbb{V}$ denote the (finite !!!) set of constant right-hand sides. Then variables may only take values from $\mathbb{V}^\top$:-))

→ The occurring effects can be taken from

$$\mathbb{D}_f \to \mathbb{D}_f \qquad \text{with} \qquad \mathbb{D}_f = (\mathit{Vars} \to \mathbb{V}^\top)_\perp$$

→ The complete lattice is huge, but finite !!!

Improvement:

$\longrightarrow$ Not all functions from $\mathbb{D}_f \to \mathbb{D}_f$ will occur :-)

$\longrightarrow$ All occurring functions $\lambda D.\bot \neq M$ are of the form:

$$\begin{array}{lcll} M & = & \{x \mapsto (b_x \sqcup \bigsqcup_{y \in I_x} y) \mid x \in \mathit{Vars}\} & \text{where:} \\ M\ D & = & \{x \mapsto (b_x \sqcup \bigsqcup_{y \in I_x} D\ y) \mid x \in \mathit{Vars}\} & \text{für} \quad D \neq \bot \end{array}$$

$\longrightarrow$ Let $\mathbb{M}$ denote the set of all these functions. Then for $M_1, M_2 \in \mathbb{M}$ $(M_1 \neq \lambda\, D.\, \bot \neq M_2)$:

$$(M_1 \sqcup M_2)\ x \quad = \quad (M_1\ x) \sqcup (M_2\ x)$$

$\longrightarrow$ For $k = \#\mathit{Vars}$, $\mathbb{M}$ has height $\mathcal{O}(k^2)$:-)

Improvement (Cont.):

→ Also, composition can be directly implemented:

$$\begin{array}{rcll} (M_1 \circ M_2)\; x & = & b' \sqcup \bigsqcup_{y \in I'} y & \quad\text{with} \\ b' & = & b \sqcup \bigsqcup_{z \in I} b_z & \\ I' & = & \bigcup_{z \in I} I_z & \quad\text{where} \\ M_1\; x & = & b \sqcup \bigsqcup_{y \in I} y & \\ M_2\; z & = & b_z \sqcup \bigsqcup_{y \in I_z} y & \end{array}$$

→ The effects of assignments then are:

$$[\![x = e;]\!]^\sharp \;=\; \begin{cases} \mathsf{Id}_{Vars} \oplus \{x \mapsto c\} & \text{if} \quad e = c \in \mathbb{Z} \\ \mathsf{Id}_{Vars} \oplus \{x \mapsto y\} & \text{if} \quad e = y \in Vars \\ \mathsf{Id}_{Vars} \oplus \{x \mapsto \top\} & \text{otherwise} \end{cases}$$

... in the Example:

$$
\begin{array}{lll}
[\![t = 0;]\!]^\sharp & = & \{a_1 \mapsto a_1, \mathsf{ret} \mapsto \mathsf{ret}, \boxed{t \mapsto 0}\} \\
[\![a_1 = t;]\!]^\sharp & = & \{\boxed{a_1 \mapsto t}, \mathsf{ret} \mapsto \mathsf{ret}, t \mapsto t\}
\end{array}
$$

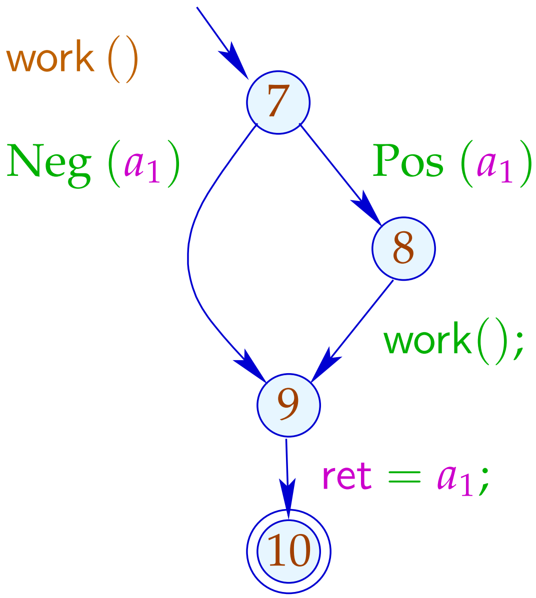
In order to implement the analysis, we additionally must construct the effect of a call $k = (\_, f\,();, \_)$ from the effect of a procedure f :

$$
\begin{array}{lll}
[\![k]\!]^\sharp & = & H\,([\![f]\!]^\sharp) \qquad \text{where:} \\
H\,(M) & = & \mathsf{Id}|_{Locals} \oplus \{\mathsf{x} \mapsto (M \circ \mathsf{enter}^\sharp)|_{Globals} \\
\mathsf{enter}^\sharp\; x & = & \begin{cases} x & \text{if} \quad x \in Globals \\ 0 & \text{otherwise} \end{cases}
\end{array}
$$

... in the Example:

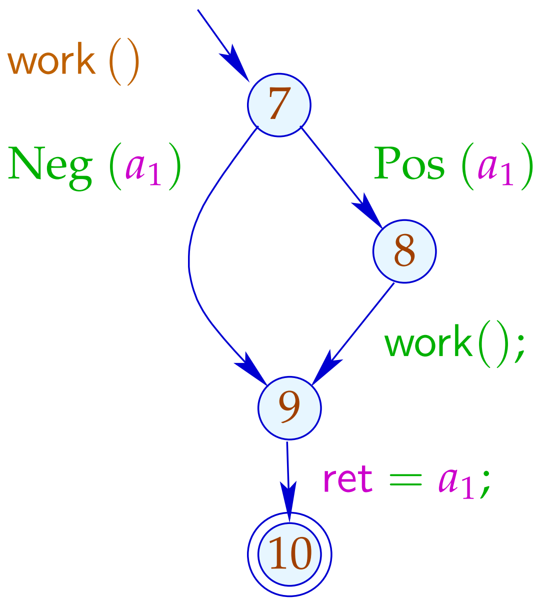
$$\begin{array}{lrcl} \text{If} & [\![\text{work}]\!]^\sharp & = & \{a_1 \mapsto a_1, \text{ret} \mapsto a_1, t \mapsto t\} \\ \text{then} & H\,[\![\text{work}]\!]^\sharp & = & \text{Id}_{\{t\}} \oplus \{a_1 \mapsto a_1, \text{ret} \mapsto a_1\} \\ & & = & \{a_1 \mapsto a_1, \text{ret} \mapsto a_1, t \mapsto t\} \end{array}$$

Now we can perform fixpoint iteration :-)



	1
7	$\{a_1 \mapsto a_1, \mathsf{ret} \mapsto \mathsf{ret}, t \mapsto t\}$
9	$\{a_1 \mapsto a_1, \mathsf{ret} \mapsto \mathsf{ret}, t \mapsto t\}$
10	$\{a_1 \mapsto a_1, \mathsf{ret} \mapsto a_1, t \mapsto t\}$
8	$\{a_1 \mapsto a_1, \mathsf{ret} \mapsto \mathsf{ret}, t \mapsto t\}$

$$\begin{array}{lll} [\![(8,\ldots,9)]\!]^\sharp \circ [\![8]\!]^\sharp & = & \{a_1 \mapsto a_1, \mathsf{ret} \mapsto a_1, t \mapsto t\} \;\circ \\ & & \{a_1 \mapsto a_1, \mathsf{ret} \mapsto \mathsf{ret}, t \mapsto t\} \\ & = & \{a_1 \mapsto a_1, \mathsf{ret} \mapsto a_1, t \mapsto t\} \end{array}$$



	2
7	$\{a_1 \mapsto a_1, \mathsf{ret} \mapsto \mathsf{ret}, t \mapsto t\}$
9	$\boxed{\{a_1 \mapsto a_1, \mathsf{ret} \mapsto a_1 \sqcup \mathsf{ret}, t \mapsto t\}}$
10	$\boxed{\{a_1 \mapsto a_1, \mathsf{ret} \mapsto a_1, t \mapsto t\}}$
8	$\{a_1 \mapsto a_1, \mathsf{ret} \mapsto \mathsf{ret}, t \mapsto t\}$

$$
\begin{aligned}
[\![(8,\ldots,9)]\!]^\sharp \circ [\![8]\!]^\sharp &= \{a_1 \mapsto a_1, \mathsf{ret} \mapsto a_1, t \mapsto t\} \circ \\
&\quad\ \{a_1 \mapsto a_1, \mathsf{ret} \mapsto \mathsf{ret}, t \mapsto t\} \\
&= \{a_1 \mapsto a_1, \mathsf{ret} \mapsto a_1, t \mapsto t\}
\end{aligned}
$$