Let $(\rho, \mu) \in State_h$. Then we obtain for the new edges:

$$
\begin{array}{lcl}
[\![x = \mathsf{new}();]\!]\ (\rho, \mu) & = & (\rho \oplus \{x \mapsto \mathsf{ref}\, h\}, \\
 & & \quad \mu \oplus \{(\mathsf{ref}\, h, i) \mapsto 0, (i \in \mathbb{N}_0) \\
[\![x = y[e];]\!]\ (\rho, \mu) & = & (\rho \oplus \{x \mapsto \mu\,(\rho\, y, [\![e]\!]\, \rho)\}, \mu) \\
[\![y[e_1] = e_2;]\!]\ (\rho, \mu) & = & (\rho, \mu \oplus \{(\rho\, y, [\![e_1]\!]\, \rho) \mapsto [\![e_2]\!]\, \rho\})
\end{array}
$$

Warning:

This semantics is too detailled in that it computes with absolute Addresses. Accordingly, the two programs:

$$x = \mathsf{new}();\qquad\qquad y = \mathsf{new}();$$
$$y = \mathsf{new}();\qquad\qquad x = \mathsf{new}();$$

are not considered as equivalent !!?

Possible Solution:

Define equivalence only up to permutation of addresses :-)

Alias Analysis 1. Idea:

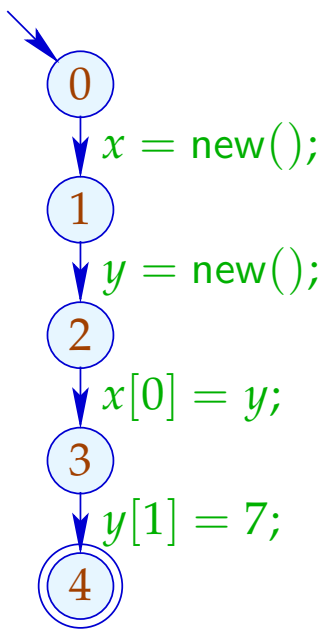
- Distinguish finitely many classes of blocks.
- Collect all addresses of a block into one set!
- Use sets of addresses as abstract values!

$\Longrightarrow$ Points-to-Analysis

$$
\begin{array}{lcll}
Addr^\sharp & = & Edges & // \text{ creation edges} \\
Val^\sharp & = & 2^{Addr^\sharp} & // \text{ abstract values} \\
Store^\sharp & = & Addr^\sharp \to Val^\sharp & // \text{ abstract store} \\
State^\sharp & = & (Vars \to Val^\sharp) \times Store^\sharp & // \text{ abstract states}
\end{array}
$$

// complete lattice !!!

... in the Simple Example:



	x	y	$(0,1)$
0	$\top$	$\top$	$\emptyset$
1	$\{(0,1)\}$	$\top$	$\emptyset$
2	$\{(0,1)\}$	$\{(1,2)\}$	$\emptyset$
3	$\{(0,1)\}$	$\{(1,2)\}$	$\{(1,2)\}$
4	$\{(0,1)\}$	$\{(1,2)\}$	$\{(1,2)\}$

The Effects of Edges:

$$
\begin{array}{lcl}
[\![(\_, ;, \_)]\!]^\sharp\,(D, M) & = & (D, M) \\
[\![(\_, \mathrm{Pos}(e), \_)]\!]^\sharp\,(D, M) & = & (D, M) \\
[\![(\_, x = y;, \_)]\!]^\sharp\,(D, M) & = & (D \oplus \{x \mapsto D\,y\}, M) \\
[\![(\_, x = e;, \_)]\!]^\sharp\,(D, M) & = & (D \oplus \{x \mapsto \emptyset\}, M) \qquad , \qquad e \notin \mathit{Vars} \\
 & & \\
[\![(u, x = \mathsf{new}();, v)]\!]^\sharp\,(D, M) & = & (D \oplus \{x \mapsto \{(u, v)\}\}, M) \\
[\![(\_, x = y[e];, \_)]\!]^\sharp\,(D, M) & = & (D \oplus \{x \mapsto \bigcup\{M(f) \mid f \in D\,y\}\}, M) \\
[\![(\_, y[e_1] = x;, \_)]\!]^\sharp\,(D, M) & = & (D, M \oplus \{f \mapsto (M\,f \cup D\,x) \mid f \in D\,y\})
\end{array}
$$

Warning:

- The value Null has been ignored. Dereferencing of Null or negative indices are not detected :-(
- Destructive updates are only possible for variables, not for blocks in storage!

  $\Longrightarrow$ no information, if not all block entries are initialized before use :-((
- The effects now depend on the edge itself.

  The analysis cannot be proven correct w.r.t. the reference semantics :-(

  In order to prove correctness, we first instrument the concrete semantics with extra information which records where a block has been created.

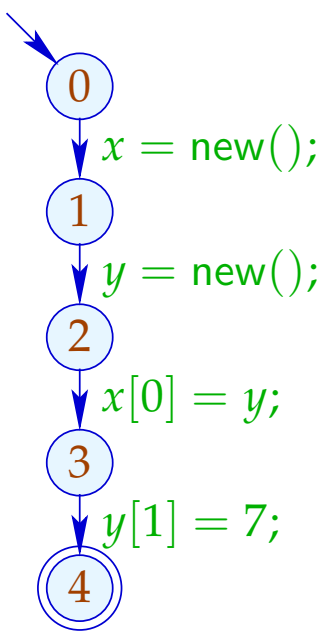
...

- We compute possible points-to information.
- From that, we can extract may-alias information.
- The analysis can be rather expensive — without finding very much :-(
- Separate information for each program point can perhaps be abandoned ??

Alias Analysis 2. Idea:

Compute for each variable and address a value which safely approximates the values at every program point simultaneously !

... in the Simple Example:



x	$\{(0,1)\}$
y	$\{(1,2)\}$
$(0,1)$	$\{(1,2)\}$
$(1,2)$	$\emptyset$

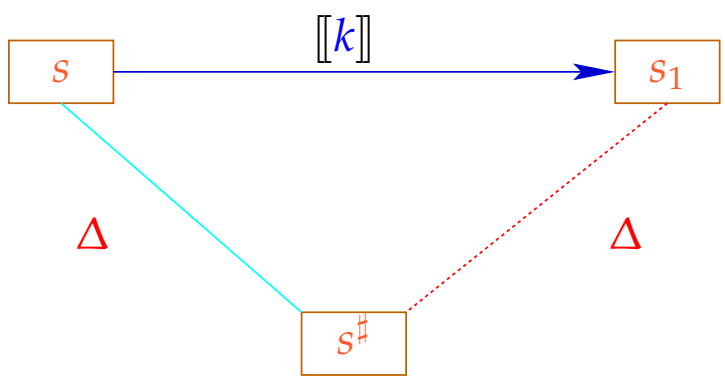
Each edge (u, lab, v) gives rise to constraints:

lab	*Constraint*
$x = y;$	$\mathcal{P}[x] \supseteq \mathcal{P}[y]$
$x = \mathsf{new}();$	$\mathcal{P}[x] \supseteq \{(u, v)\}$
$x = y[e];$	$\mathcal{P}[x] \supseteq \bigcup\{\mathcal{P}[f] \mid f \in \mathcal{P}[y]\}$
$y[e_1] = x;$	$\mathcal{P}[f] \supseteq (f \in \mathcal{P}[y])\,?\,\mathcal{P}[x] : \emptyset$ for all $f \in Addr^\sharp$

Other edges have no effect :-)

Discussion:

- The resulting constraint system has size $\mathcal{O}(k \cdot n)$ for k abstract addresses and n edges :-(
- The number of necessary iterations is $\mathcal{O}(k + \#Vars)$ ...
- The computed information is perhaps still too zu precise !!?
- In order to prove correctness of a solution $s^\sharp \in States^\sharp$ we show:

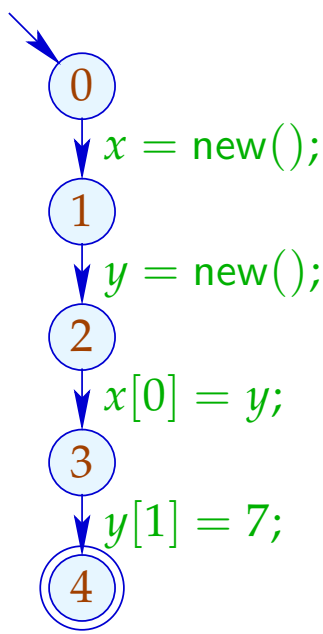


Alias Analysis

3. Idea:

Determine one equivalence relation $\equiv$ on variables x and memory accesses $y[\,]$ with $s_1 \equiv s_2$ whenever s_1, s_2 may contain the same address at some u_1, u_2

... in the Simple Example:



$$\equiv \;=\; \{\{x\},\ \{y, x[\,]\},\ \{y[\,]\}\}$$

Discussion:

- $\rightarrow$ We compute a single information fo the whole program.
- $\rightarrow$ The computation of this information maintains partitions $\pi = \{P_1, \ldots, P_m\}$:-)
- $\rightarrow$ Individual sets P_i are identified by means of representatives $p_i \in P_i$.
- $\rightarrow$ The operations on a partition π are:

$$
\begin{array}{lcll}
\mathsf{find}\,(\pi, p) & = & p_i & \text{if } p \in P_i \\
 & // & \text{returns the representative} & \\
\mathsf{union}\,(\pi, p_{i_1}, p_{i_2}) & = & \{P_{i_1} \cup P_{i_2}\} \cup \{P_j \mid i_1 \neq j \neq i_2\} & \\
 & // & \text{unions the represented classes} &
\end{array}
$$

→ If $x_1, x_2 \in Vars$ are equivalent, then also $x_1[\,]$ and $x_2[\,]$ must be equivalent :-)

→ If $P_i \cap Vars \neq \emptyset$, then we choose $p_i \in Vars$. Then we can apply union recursively :

$$
\begin{array}{lcl}
\mathsf{union}^*\,(\pi, q_1, q_2) & = & \mathsf{let}\;\; p_{i_1} = \mathsf{find}\,(\pi, q_1) \\
 & & \phantom{\mathsf{let}\;\;} p_{i_2} = \mathsf{find}\,(\pi, q_2) \\
 & & \mathsf{in}\;\; \mathsf{if}\; p_{i_1} == p_{i_2}\; \mathsf{then}\; \pi \\
 & & \phantom{\mathsf{in}\;\;} \mathsf{else}\;\; \mathsf{let}\; \pi = \mathsf{union}\,(\pi, p_{i_1}, p_{i_2}) \\
 & & \phantom{\mathsf{in}\;\;\mathsf{else}\;\;} \mathsf{in}\;\; \mathsf{if}\; p_{i_1}, p_{i_2} \in Vars\; \mathsf{then} \\
 & & \phantom{\mathsf{in}\;\;\mathsf{else}\;\;\mathsf{in}\;\;} \mathsf{union}^*\,(\pi, p_{i_1}[\,], p_{i_2}[\,])
\end{array}
$$

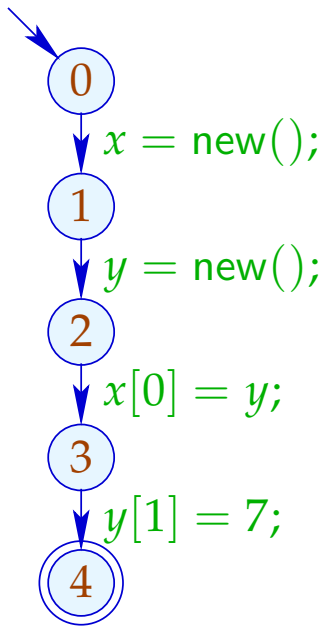
The analysis iterates over all edges once:

$$\pi = \{\{x\}, \{x[\,]\} \mid x \in \mathit{Vars}\};$$
$$\textsf{forall} \quad k = (\_, \mathit{lab}, \_) \quad \textsf{do} \quad \pi = [\![\mathit{lab}]\!]^\sharp\, \pi;$$

where:

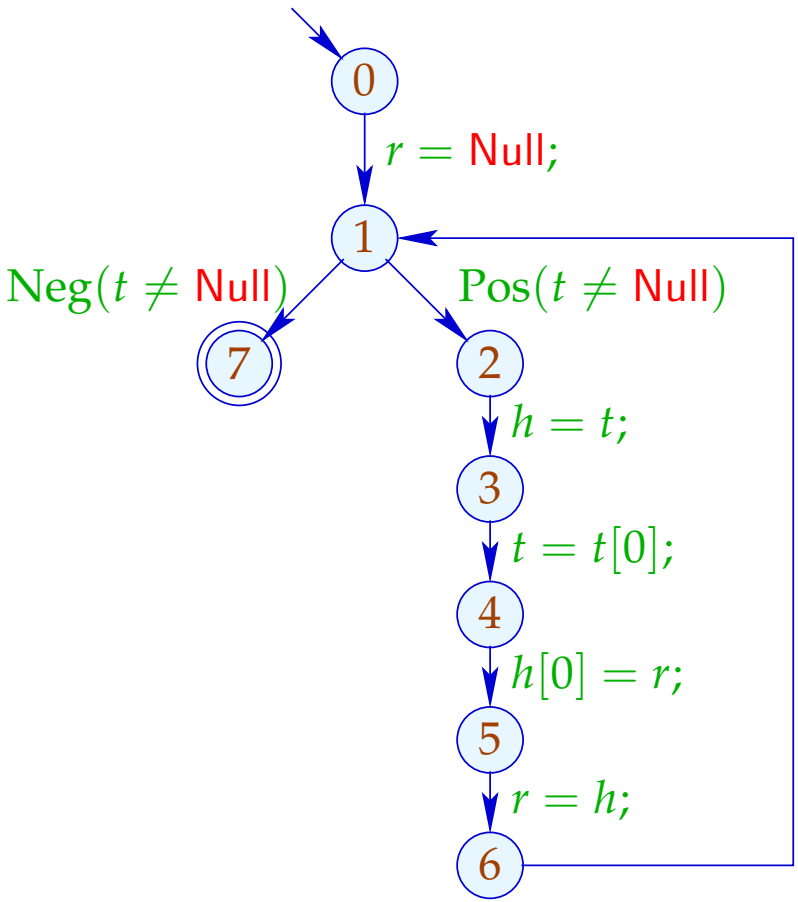
$$\begin{array}{lcll}
[\![x = y;]\!]^\sharp\, \pi & = & \textsf{union}^*\,(\pi, x, y) & \\
[\![x = y[e];]\!]^\sharp\, \pi & = & \textsf{union}^*\,(\pi, x, y[\,]) & \\
[\![y[e] = x;]\!]^\sharp\, \pi & = & \textsf{union}^*\,(\pi, x, y[\,]) & \\
[\![\mathit{lab}]\!]^\sharp\, \pi & = & \pi & \text{otherwise}
\end{array}$$

... in the Simple Example:



	$\{\{x\}, \{y\}, \{x[\,]\}, \{y[\,]\}\}$
$(0,1)$	$\{\{x\}, \{y\}, \{x[\,]\}, \{y[\,]\}\}$
$(1,2)$	$\{\{x\}, \{y\}, \{x[\,]\}, \{y[\,]\}\}$
$(2,3)$	$\{\{x\}, \boxed{\{y, x[\,]\}}, \{y[\,]\}\}$
$(3,4)$	$\{\{x\}, \{y, x[\,]\}, \{y[\,]\}\}$

... in the More Complex Example:



	$\{\{h\},\{r\},\{t\},\{h[\,]\},\{t[\,]\}\}$
$(2,3)$	$\{\boxed{\{h,t\}},\{r\},\boxed{\{h[\,],t[\,]\}}\}$
$(3,4)$	$\{\boxed{\{h,t,h[\,],t[\,]\}},\{r\}\}$
$(4,5)$	$\{\boxed{\{h,t,r,h[\,],t[\,]\}}\}$
$(5,6)$	$\{\{h,t,r,h[\,],t[\,]\}\}$

Warning:

In order to find something, we must assume that variables / addresses always receive a value before they are accessed.

Complexity:

we havve:

$\mathcal{O}(\#\,edges + \#\,Vars)$	calls of	union*
$\mathcal{O}(\#\,edges + \#\,Vars)$	calls of	find
$\mathcal{O}(\#\,Vars)$	calls of	union

$\Longrightarrow$ We require efficient Union-Find data-structure :-)