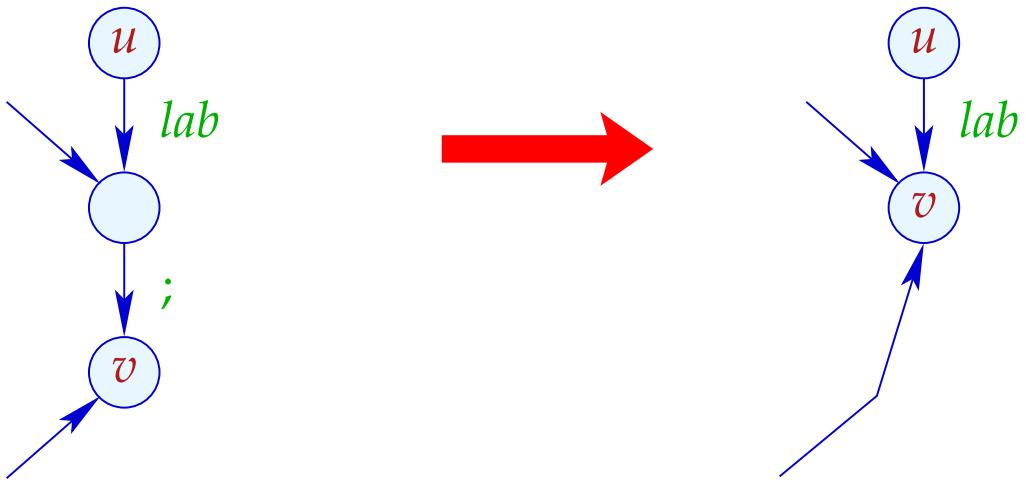
Important Subproblem: *nop*-Optimization



→ If $(v_1, ;, v)$ is an edge, v_1 has no further out-going edge.

→ Consequently, we can identify v_1 and v :-)

→ The ordering of the identifications does not matter :-))

Implementation:

- We construct a function $\mathsf{next} : \mathit{Nodes} \to \mathit{Nodes}$ with:

$$\mathsf{next}\ u = \begin{cases} \mathsf{next}\ v & \text{if} \quad (u, ;, v) \quad \text{edge} \\ u & \text{otherwise} \end{cases}$$

  Warning: This definition is only recursive if there are ;-loops ???

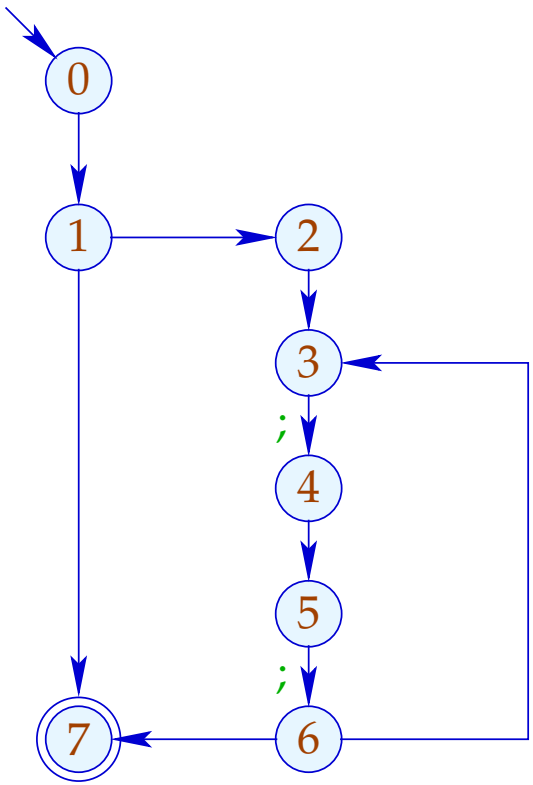
- We replace every edge:

$$(u, lab, v) \quad \Longrightarrow \quad (u, lab, \mathsf{next}\ v)$$

  ... whenever $lab \neq ;$

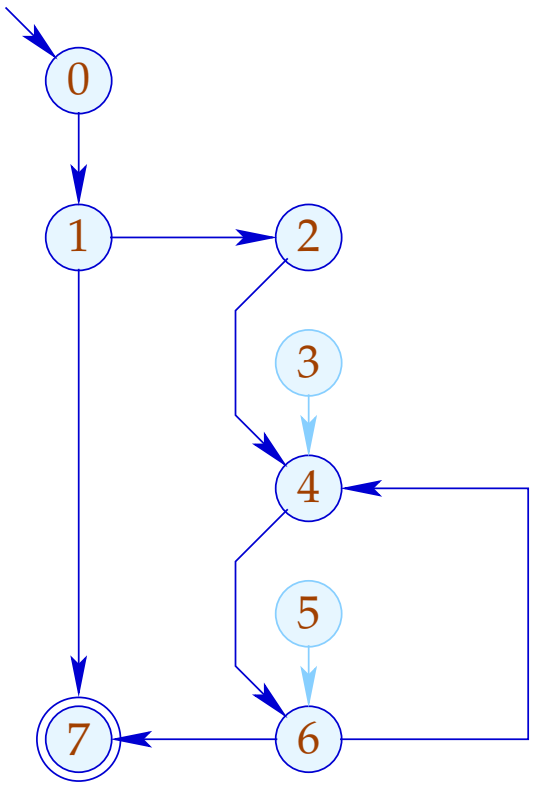
- All ;-edges are removed ;-)

Example:



$$\begin{aligned} \text{next } 1 &= 1 \\ \text{next } 3 &= 4 \\ \text{next } 5 &= 6 \end{aligned}$$

Example:



$$\begin{aligned} \text{next } 1 &= 1 \\ \text{next } 3 &= 4 \\ \text{next } 5 &= 6 \end{aligned}$$

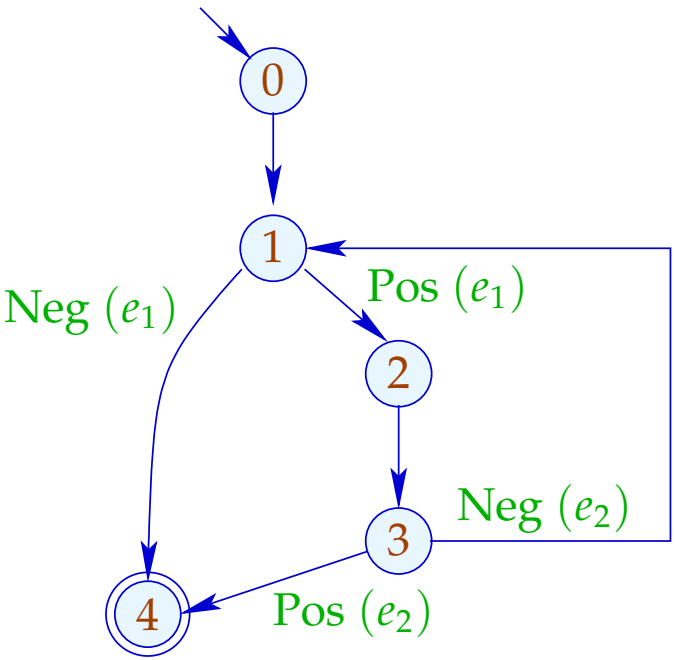
2. Subproblem: Linearization

After optimization, the CFG must again be brought into a linearly arrangement of instructions :-)

Warning:

Not every linearization is equally efficient !!!

Example:



0:

1: if (e_1) goto 2;

4: halt

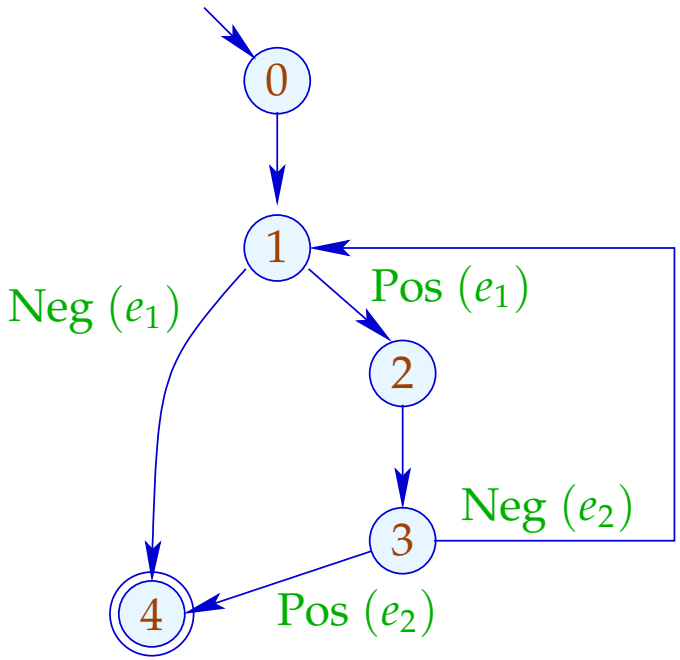
2: Rumpf

3: if (e_2) goto 4;

goto 1;

Bad: The loop body is jumped into :-(

Example:




```
0:
1:    if (!e1) goto 4;
2:    Rumpf
3:    if (!e2) goto 1;
4:    halt
```

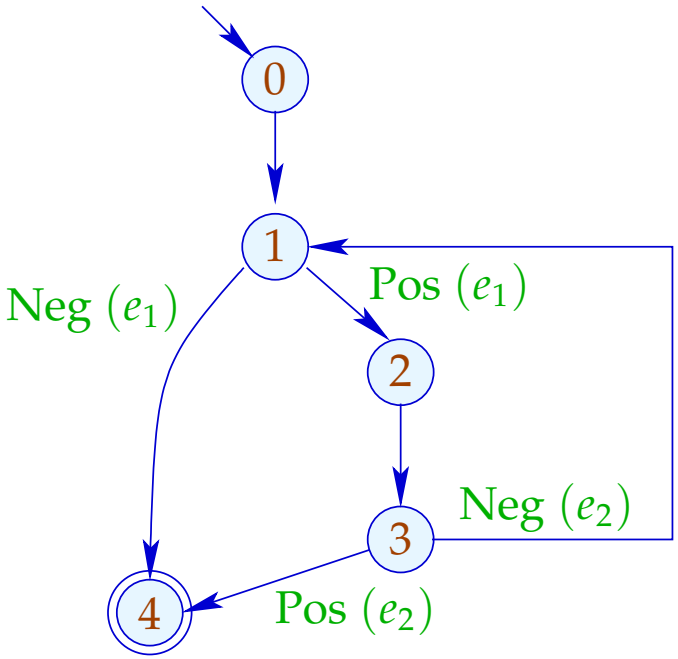
// better cache behavior :-)

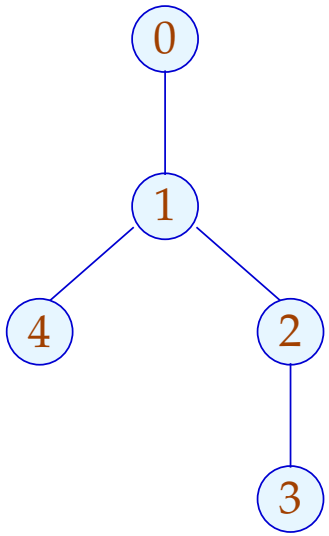
Idea:

- Assign to each node a temperature!
- always jumps to
  - (1) nodes which have already been handled;
  - (2) colder nodes.
- Temperature $\approx$ nesting-depth

For the computation, we use the pre-dominator tree and strongly connected components ...

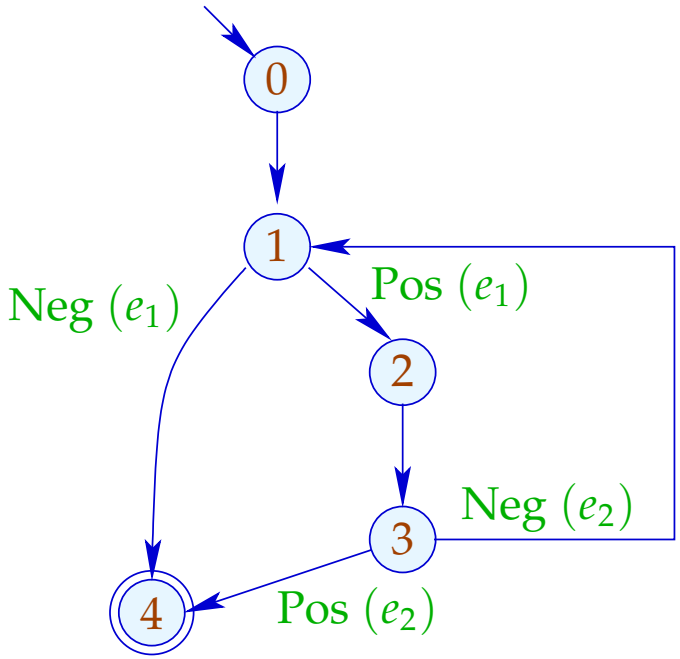
... in the Example:

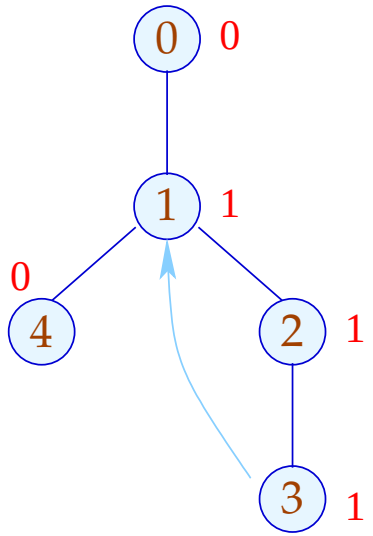




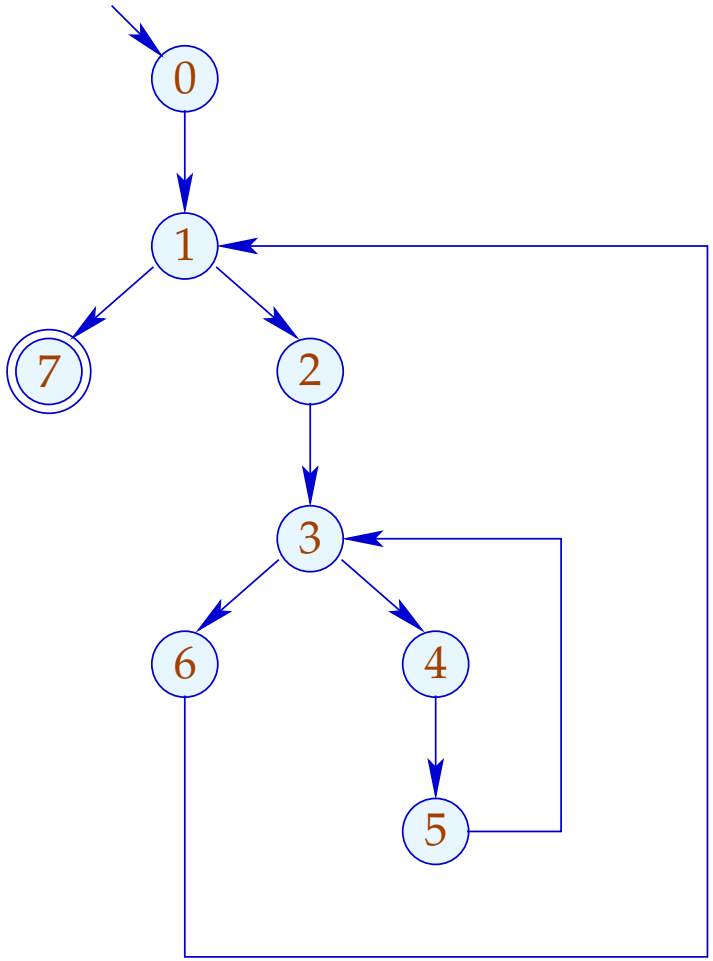
The sub-tree with back edge is hotter ...

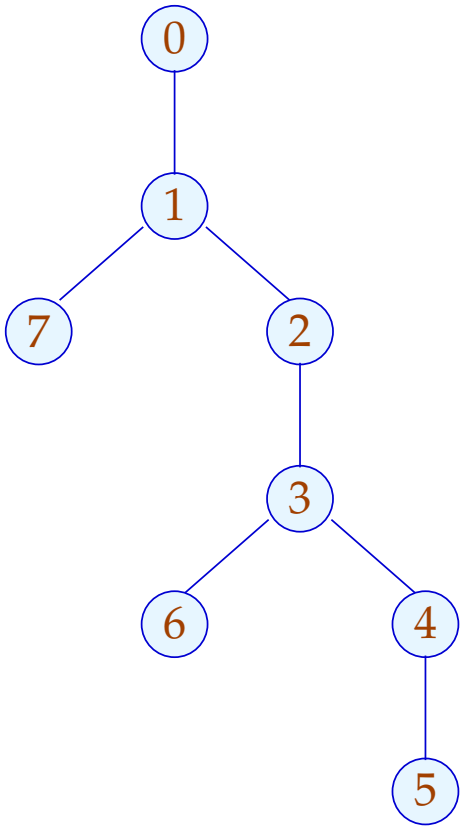
... in the Example:





More Complicated Example:





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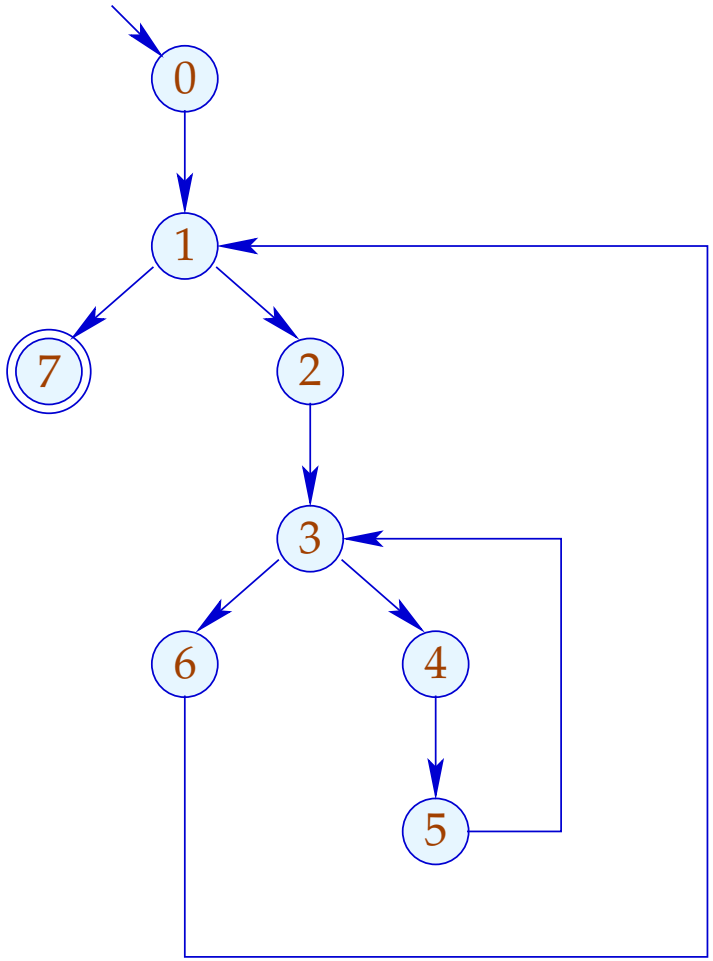

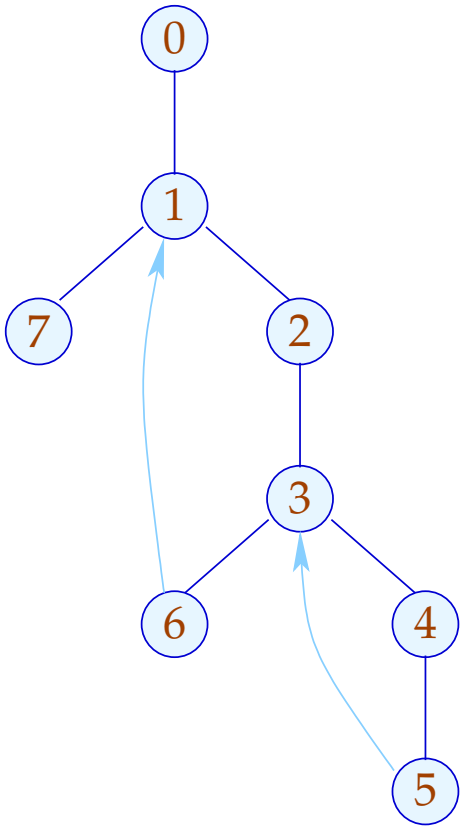

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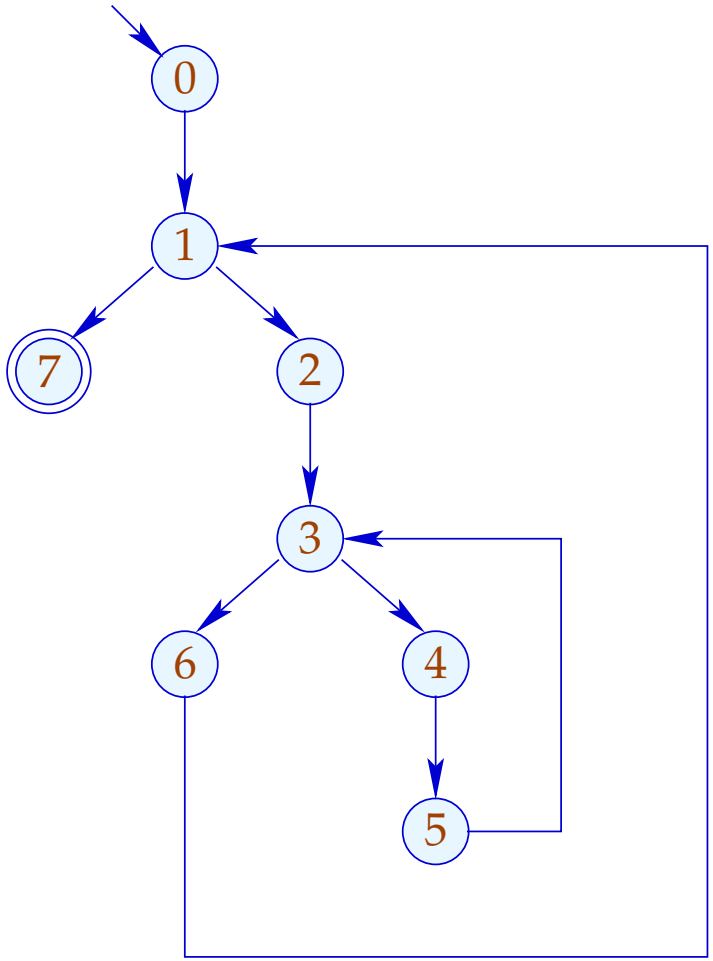
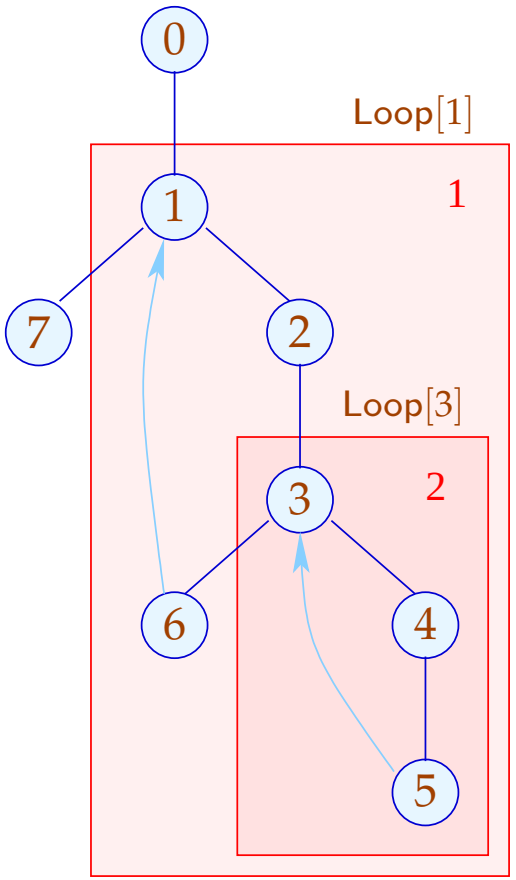
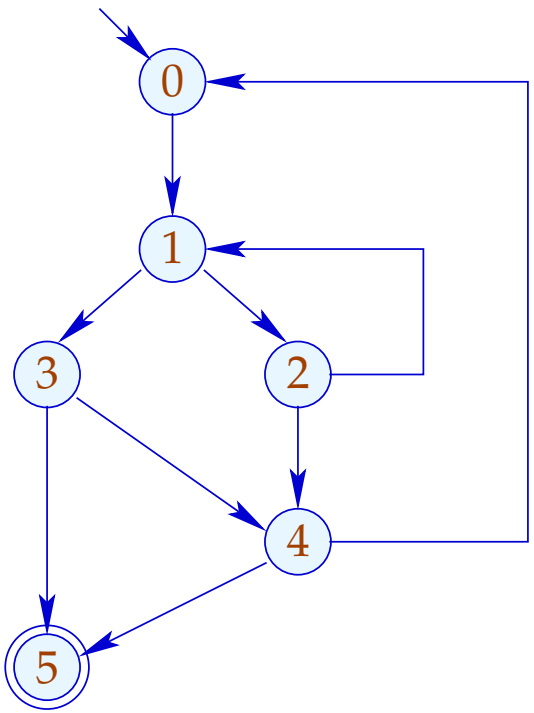
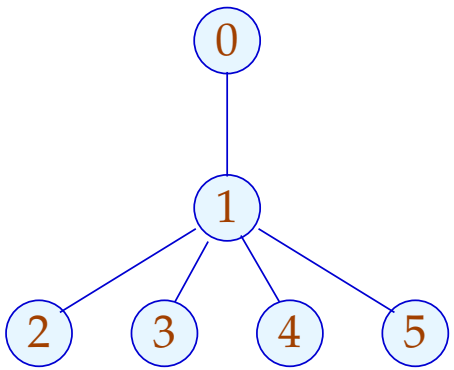




Our definition of Loop implies that (detected) loops are necessarily nested :-)

Is is also meaningful for do-while-loops with breaks ...





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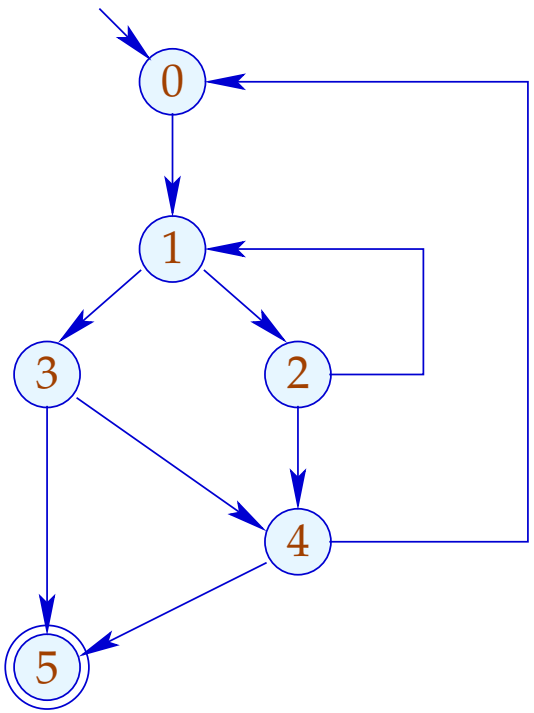
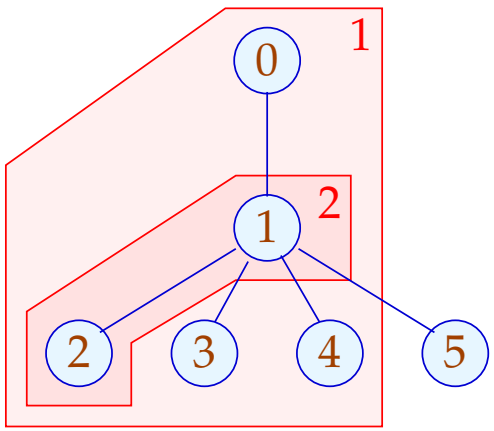




Summary: The Approach

(1) For every node, determine a temperature;

(2) Pre-order-DFS over the CFG;

- → If an edge leads to a node we already have generated code for, then we insert a jump.
- → If a node has two successors with different temperature, then we insert a jump to the colder of the two.
- → If both successors are equally warm, then it does not matter ;-)

2.3 Procedures

We extend our mini-programming language by procedures without parameters and procedure calls.

For that, we introduce a new statement:

$$f\,();$$

Every procedure f has a definition:

$$f\,()\ \{\ stmt^*\ \}$$

Additionally, we distinguish between global and local variables.

Program execution starts with the call of a procedure $\mathsf{main}\,()$.

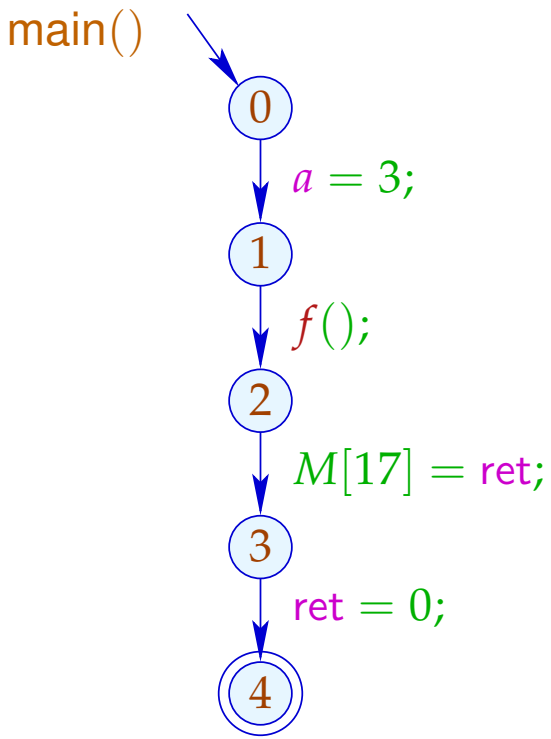
Example:

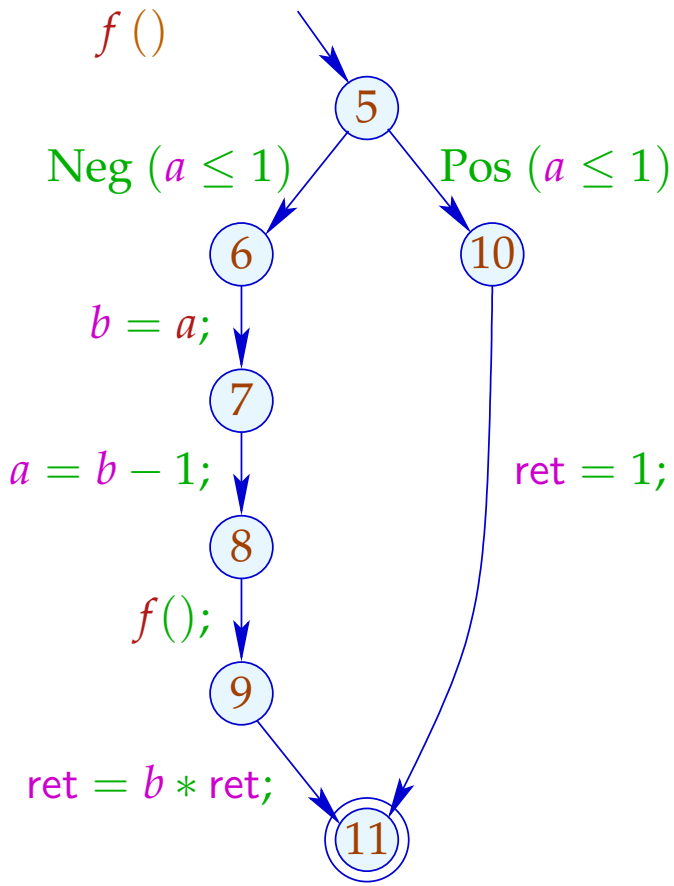
```
int a, ret;
main () {
        a = 3;
        f();
        M[17] = ret;
        ret = 0;
}
```

```
f () {
        int b;
        if (a ≤ 1) {ret = 1; goto exit;}
        b = a;
        a = b − 1;
        f();
        ret = b · ret;
exit :
}
```

Such programs can be represented by a set of CFGs: one for each procedure ...

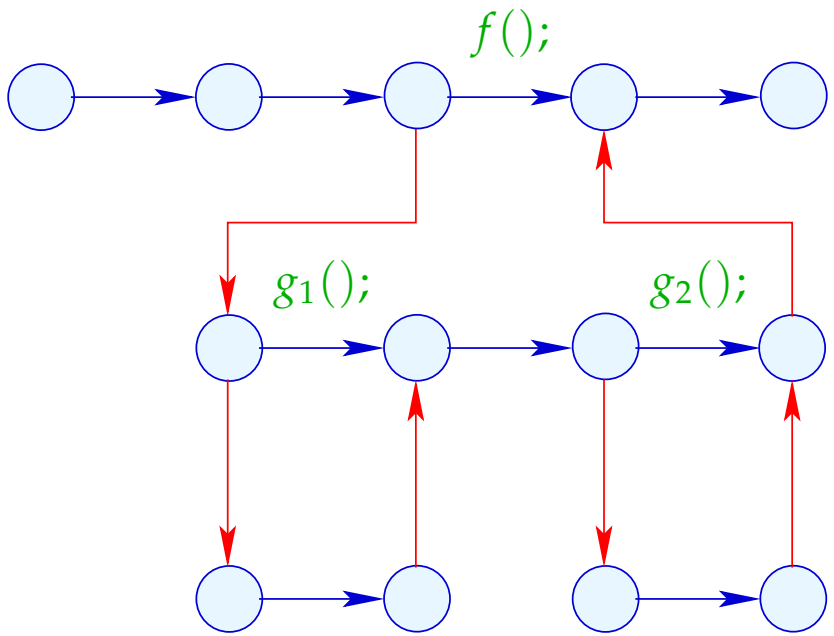
... in the Example:



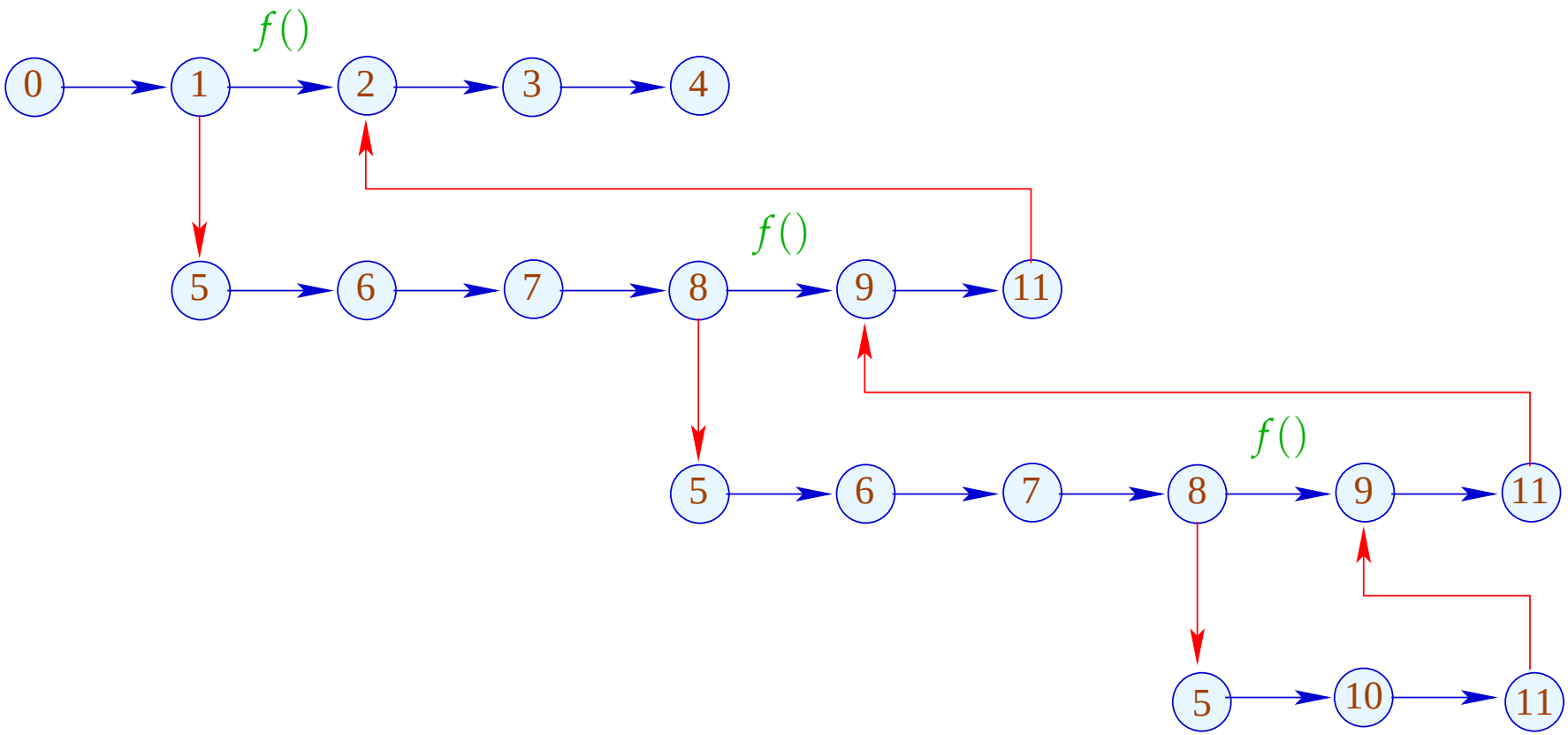


In order to optimize such programs, we require an extended operational semantics ;-)

Program executions are no longer paths, but forests:



... in the Example:



The function $[\![.]\!]$ is extended to computation forests: w :

$$[\![w]\!] : (\mathit{Vars} \to \mathbb{Z}) \times (\mathbb{N} \to \mathbb{Z}) \to (\mathit{Vars} \to \mathbb{Z}) \times (\mathbb{N} \to \mathbb{Z})$$

For a call $k = (u, f();, v)$ we must:

- determine the initial values for the locals:

$$\mathsf{enter}\ \rho = \{x \mapsto 0 \mid x \in \mathit{Locals}\} \oplus (\rho|_{\mathit{Globals}})$$

- ... combine the new values for the globals with the old values for the locals:

$$\mathsf{combine}\ (\rho_1, \rho_2) = (\rho_1|_{\mathit{Locals}}) \oplus (\rho_2|_{\mathit{Globals}})$$

- ... evaluate the computation forest inbetween:

$$\begin{array}{rcl} [\![k\ \langle w \rangle]\!]\ (\rho, \mu) & = & \mathsf{let}\ \ (\rho_1, \mu_1) = [\![w]\!]\ (\mathsf{enter}\ \rho, \mu) \\ & & \mathsf{in}\ \ \ (\mathsf{combine}\ (\rho, \rho_1), \mu_1) \end{array}$$

Warning:

- In general, $[\![w]\!]$ is only partially defined :-)
- Dedicated global/local variables a_i, b_i, ret can be used to simulate specific calling conventions.
- The standard operational semantics relies on configurations which maintain a call stack.
- Computation forests are better suited for the construction of analyses and correctness proofs :-)
- It is an awkward (but useful) exercise to prove the equivalence of the two approaches ...

Configurations:

$$
\begin{array}{lcl}
\mathit{configuration} & == & \mathit{stack} \times \mathit{store} \\
\mathit{store} & == & \mathit{globals} \times (\mathbb{N} \to \mathbb{Z}) \\
\mathit{globals} & == & (\mathit{Globals} \to \mathbb{Z}) \\
\mathit{stack} & == & \mathit{frame} \cdot \mathit{frame}^* \\
\mathit{frame} & == & \mathit{point} \times \mathit{locals} \\
\mathit{locals} & == & (\mathit{Locals} \to \mathbb{Z})
\end{array}
$$

A *frame* specifies the local state of computation inside a procedure call :-)

The leftmost frame corresponds to the current call.

Computation steps refer to the current call :-)

The novel kinds of steps:

call $k = (u, f\ ();, v)$:

$$(\boxed{(u, \rho)} \cdot \sigma, \langle \gamma, \mu \rangle) \Longrightarrow (\boxed{(u_f, \{x \to 0 \mid x \in \mathit{Locals}\}) \cdot (v, \rho)} \cdot \sigma, \langle \gamma, \mu \rangle)$$

u_f entry point of f

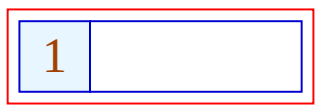
return:

$$(\boxed{(r_f, _)} \cdot \sigma, \langle \gamma, \mu \rangle) \Longrightarrow (\sigma, \langle \gamma, \mu \rangle)$$

r_f return point of f

The call stack explicitly implements the DFS traversal through the computation forest :-)

... in the Example:



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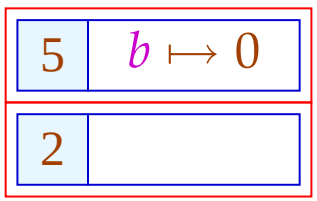

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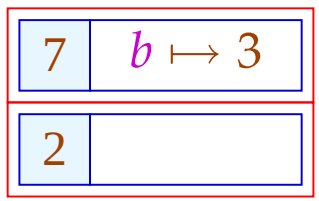

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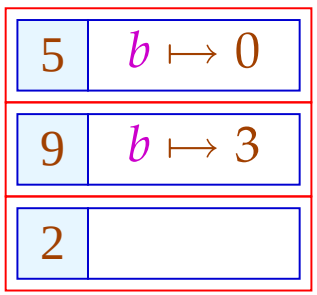

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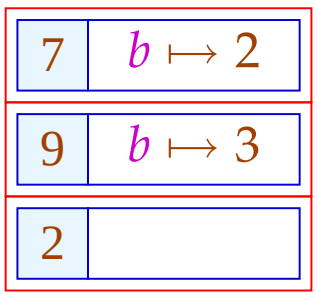

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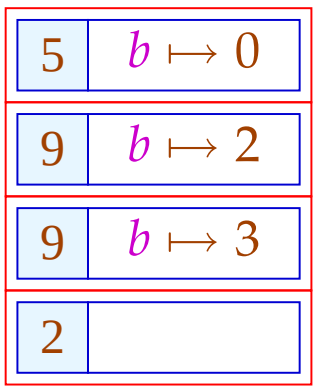

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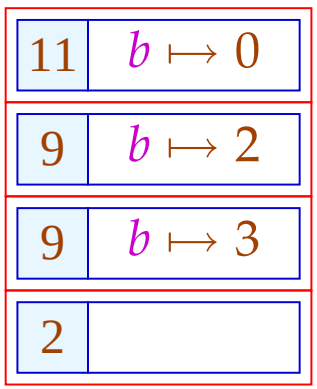

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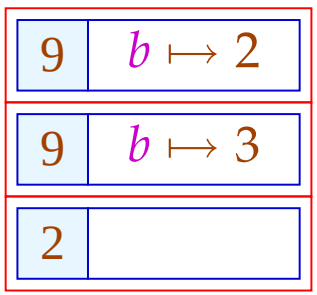

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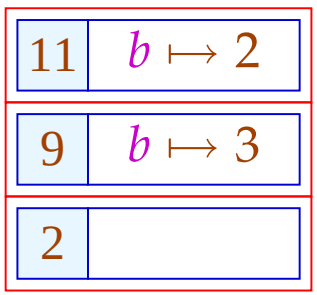

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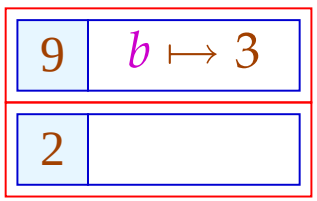

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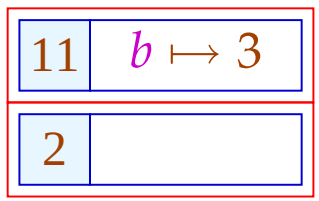

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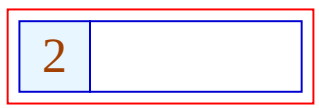

This operational semantics is quite realistic :-)

Costs for a Procedure Call:

Before entering the body:
- Creating a stack frame;
- assigning of the parameters;
- Saving the registers;
- Saving the return address;
- Jump to the body.

At procedure exit:
- Freeing the stack frame.
- Restoring the registers.
- Passing of the result.
- Return behind the call.

$\Longrightarrow$ ... quite expensive !!!

1. Idea: Inlining

Copy the procedure body at every call site !!!

Example:

```
abs () {
    a2 = −a1;
    max ();
}
```

```
max () {
    if (a1 < a2) { ret = a2; goto _exit; }
    ret = a1;
_exit :
}
```

... yields:

```
abs () {
a2 = −a1;
        if (a1 < a2) { ret = a2; goto _exit; }
        ret = a1;
 _exit :
}
```

Problems:

- The copied block may modify the locals of the calling procedure ???
- More general: Multiple use of local variable names may lead to errors.
- Multiple calls of a procedure may lead to code duplication :-((
- How can we handle recursion ???