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Fischer, Rabin,1974 : $\text{PSAT} \geq \text{NTIME}(2^{2^{c \cdot n}})$

## 3.3 Improving the Memory Layout

Goal:

- Better utilization of caches

  $\Longrightarrow$ reduction of the number of cache misses

- Reduction of allocation/de-allocation costs

  $\Longrightarrow$ replacing heap allocation by stack allocation

  $\Longrightarrow$ support to free superfluous heap objects

- Reduction of access costs

  $\Longrightarrow$ short-circuiting indirection chains (Unboxing)

## 1. Cache Optimization:

Idea: local memory access

- Loading from memory fetches not just one byte but fills a complete cache line.
- Access to neighbored cells become cheaper.
- If all data of an inner loop fits into the cache, the iteration becomes maximally memory-efficient ...

## Possible Solutions:

$\rightarrow$ Reorganize the data accesses !

$\rightarrow$ Reorganize the data !

Such optimizations can be made fully automatic only for arrays :-(

## Example:

```
for ($j = 1; j < n; j{+}{+}$)
    for ($i = 1; i < m; i{+}{+}$)
        $a[i][j] = a[i-1][j-1] + a[i][j];$
```

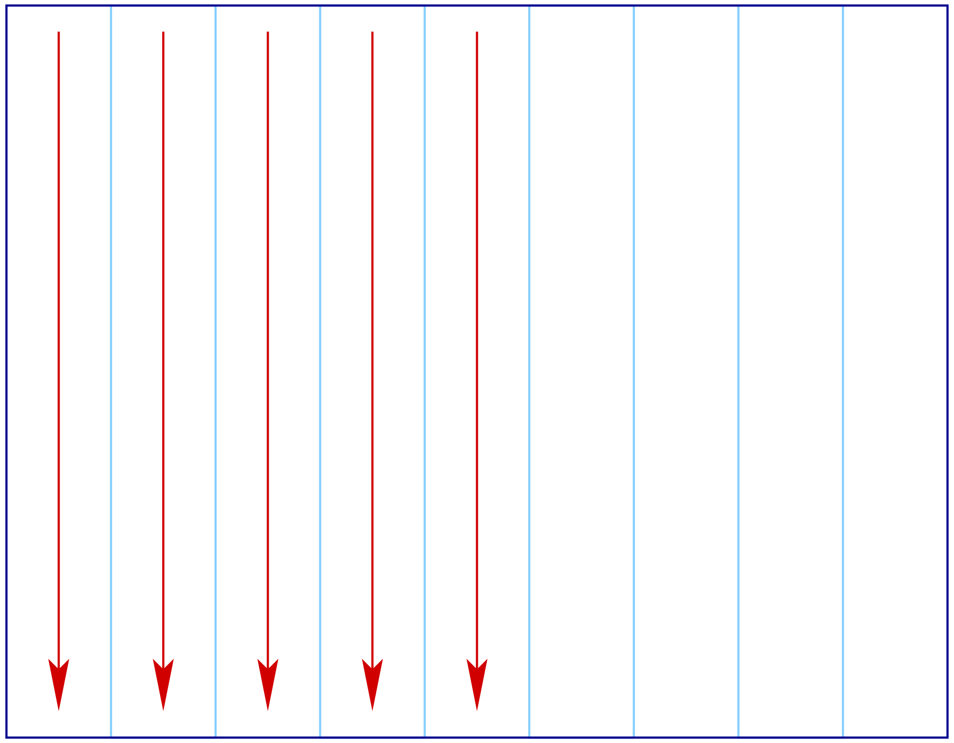
$\Longrightarrow$ At first, always iterate over the rows!

$\Longrightarrow$ Exchange the ordering of the iterations:

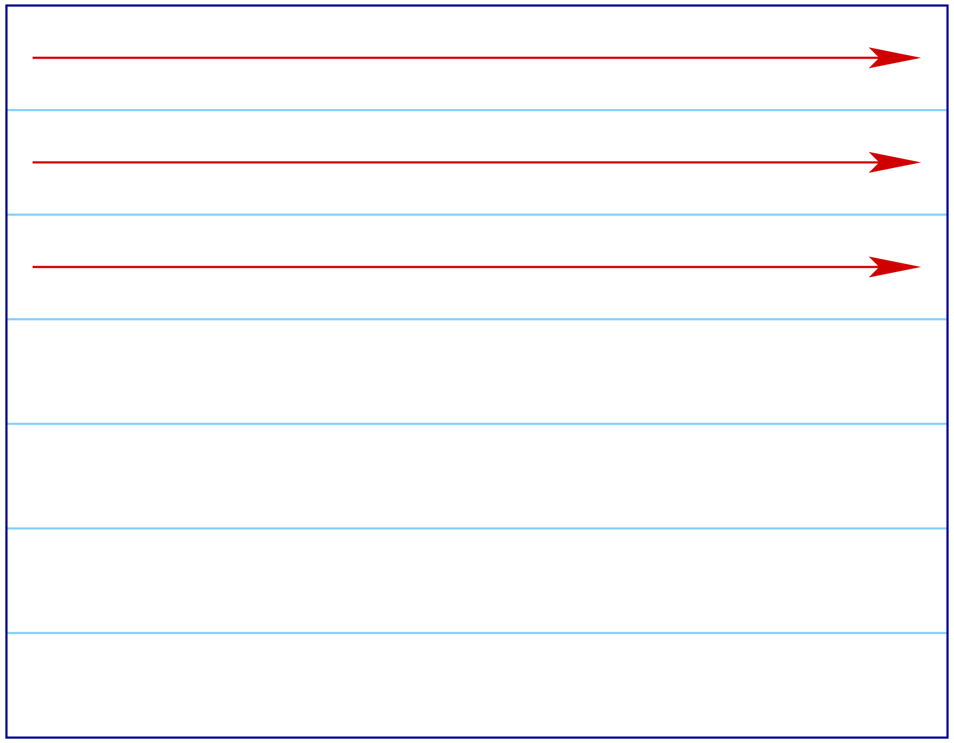
```
for $(i = 1; i < m; i{+}{+})$
    for $(j = 1; j < n; j{+}{+})$
        $a[i][j] = a[i-1][j-1] + a[i][j];$
```

When is this permitted???

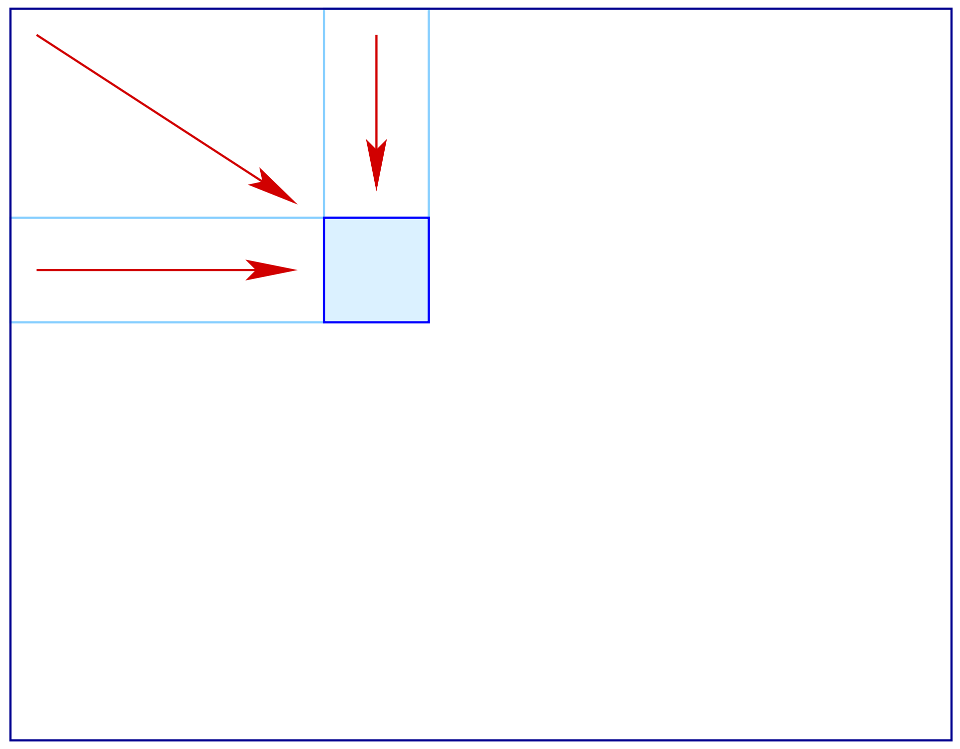
## Iteration Scheme: before:



## Iteration Scheme: after:



## Iteration Scheme: allowed dependencies:



In our case, we must check that the following equation systems have no solution:

| Write | | Read |
|---|---|---|
| $(i_1, j_1)$ | $=$ | $(i_2 - 1, j_2 - 1)$ |
| $i_1$ | $\leq$ | $i_2$ |
| $j_2$ | $\leq$ | $j_1$ |
| $(i_1, j_1)$ | $=$ | $(i_2 - 1, j_2 - 1)$ |
| $i_2$ | $\leq$ | $i_1$ |
| $j_1$ | $\leq$ | $j_2$ |

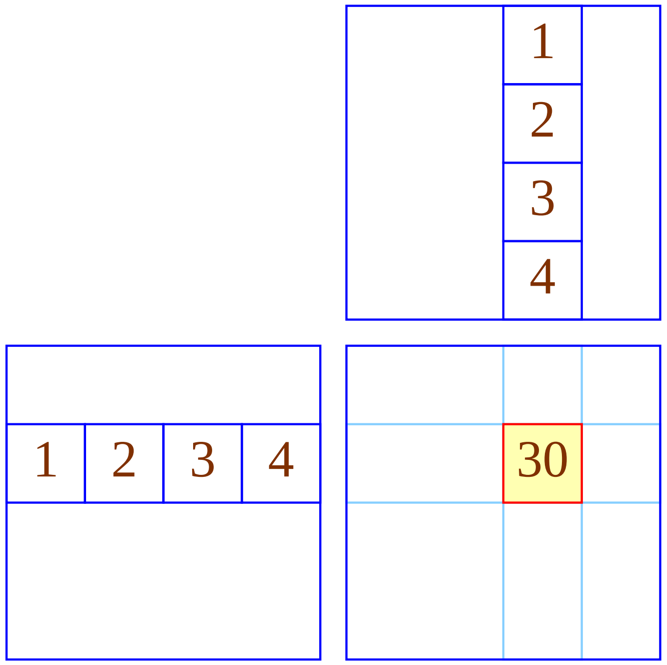
The first implies: $j_2 \leq j_2 - 1$ Hurra!

The second implies: $i_2 \leq i_2 - 1$ Hurra!

## Example: Matrix-Matrix Multiplication

```
for $(i = 0; i < N; i{+}{+})$
    for $(j = 0; j < M; j{+}{+})$
        for $(k = 0; k < K; k{+}{+})$
            $c[i][j] = c[i][j] + a[i][k] \cdot b[k][j];$
```

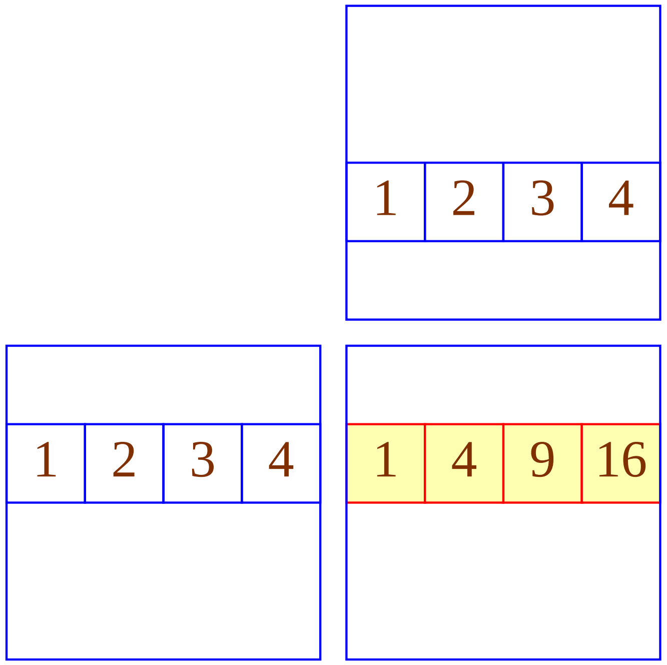
Over $b[\,][\,]$ the iteration is columnwise :-(


1
2
3
4
1
2
3
4
30

Exchange the two inner loops:

```
for $(i = 0; i < N; i{+}{+})$
    for $(k = 0; k < K; k{+}{+})$
        for $(j = 0; j < M; j{+}{+})$
            $c[i][j] = c[i][j] + a[i][k] \cdot b[k][j];$
```

Is this permitted ???


1
2
3
4
1
2
3
4
1
4
9
16

## Discussion:

- Correctness follows as before :-)
- A similar idea can also be used for the implementation of multiplication for row compressed matrices :-))
- Sometimes, the program must be massaged such that the transformation becomes applicable :-(
- Matrix-matrix multiplication perhaps requires initialization of the result matrix first ...

```
for $(i = 0; i < N; i{+}{+})$
    for $(j = 0; j < M; j{+}{+})$ {
        $c[i][j] = 0;$
        for $(k = 0; k < K; k{+}{+})$
            $c[i][j] = c[i][j] + a[i][k] \cdot b[k][j];$
    }
```

- Now, the two iterations can no longer be exchanged :-(
- The iteration over $j$, however, can be duplicated ...

```
for  $(i = 0; i < N; i{+}{+})$  {
        for  $(j = 0; j < M; j{+}{+})$  $c[i][j] = 0;$
        for  $(j = 0; j < M; j{+}{+})$
                for  $(k = 0; k < K; k{+}{+})$
                        $c[i][j] = c[i][j] + a[i][k] \cdot b[k][j];$
}
```

## Correctness:

$\Longrightarrow$ The read entries (here: no) may not be modified in the remaining body of the loop !!!

$\Longrightarrow$ The ordering of the write accesses to a memory cell may not be changed :-)

## We obtain:

```
for  $(i = 0; i < N; i{+}{+})$  {
    for  $(j = 0; j < M; j{+}{+})$  $c[i][j] = 0;$
    for  $(k = 0; k < K; k{+}{+})$
        for  $(j = 0; j < M; j{+}{+})$
            $c[i][j] = c[i][j] + a[i][k] \cdot b[k][j];$
}
```

## Discussion:

- Instead of fusing several loops, we now have distributed the loops    :-)
- Accordingly, conditionals may be moved out of the loop
  $\Longrightarrow$    if-distribution ...

## Warning:

Instead of using this transformation, the inner loop could also be optimized as follows:

```
for $(i = 0; i < N; i{+}{+})$
    for $(j = 0; j < M; j{+}{+})$ {
        $t = 0;$
        for $(k = 0; k < K; k{+}{+})$
            $t = t + a[i][k] \cdot b[k][j];$
        $c[i][j] = t;$
    }
```

## Idea:

If we find heavily used array elements $a[e_1] \ldots [e_r]$ whose index expressions stay constant within the inner loop, we could instead also provide auxiliary registers :-)

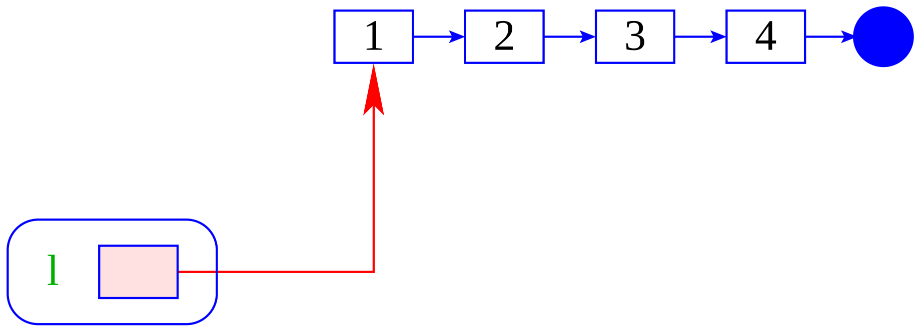
## Warning:

The latter optimization prohibits the former and vice versa ...

## Discussion:

- so far, the optimizations are concerned with iterations over arrays.
- Cache-aware organization of other data-structures is possible, but in general not fully automatic ...

## Example: Stacks



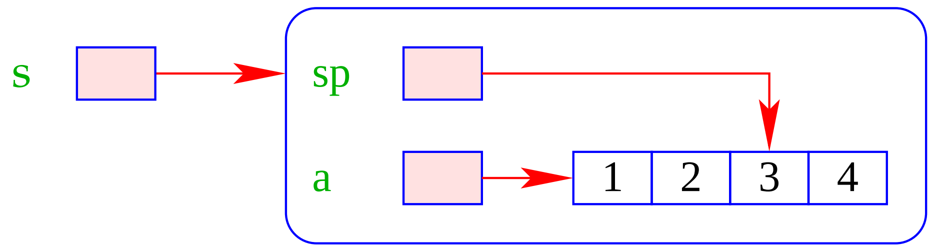
## Advantage:

+ The implementation is simple :-)
+ The operations push / pop require constant time :-)
+ The data-structure may grow arbitrarily :-)

## Disadvantage:

– The individual list objects may be arbitrarily dispersed over the memory :-(

## Alternative:



## Advantage:

+ The implementation is also simple :-)
+ The operations push / pop still require constant time :-)
+ The data are consequtively allocated; stack oscillations are typically small

$\Longrightarrow$ better Cache behavior !!!

## Disadvantage:

– The data-structure is bounded :-(

## Improvement:

- If the array is full, replace it with another of double size !!!
- If the array drops empty to a quarter, halve the array again !!!

$\Longrightarrow$ The extra amortized costs are constant :-)

$\Longrightarrow$ The implementation is no longer so trivial :-}

## Discussion:

$\rightarrow$ The same idea also works for queues :-)

$\rightarrow$ Other data-structures are attempted to organize blockwise.

Problem: how can accesses be organized such that they refer mostly to the same block ???

$\Longrightarrow$ Algorithms for external data

## 2. Stack Allocation instead of Heap Allocation

Problem:

- Programming languages such as Java allocate all data-structures in the heap — even if they are only used within the current method :-(
- If no reference to these data survives the call, we want to allocate these on the stack :-)

$\Longrightarrow$ Escape Analysis

## Idea:

Determine points-to information.

Determine if a created object is possibly reachable from the out side ...

## Example: Our Pointer Language

$$x = \mathsf{new}();$$
$$y = \mathsf{new}();$$
$$x[A] = y;$$
$$z = y;$$
$$\mathsf{ret} = z;$$

... could be a possible method body ;-)

Accessible from the outside world are memory blocks which:

- are assigned to a global variable such as ret; or
- are reachable from global variables.

## ... in the Example:

$$x = \mathsf{new}();$$
$$y = \mathsf{new}();$$
$$x[A] = y;$$
$$z = y;$$
$$\mathsf{ret} = \boxed{z};$$

Accessible from the outside world are memory blocks which:

- are assigned to a global variable such as ret; or
- are reachable from global variables.

## ... in the Example:

$$
\begin{aligned}
&x = \mathsf{new}();\\
&y = \mathsf{new}();\\
&x[A] = y;\\
&z = \boxed{y};\\
&\mathsf{ret} = \boxed{z};
\end{aligned}
$$

Accessible from the outside world are memory blocks which:

- are assigned to a global variable such as ret; or
- are reachable from global variables.

## ... in the Example:

$$x = \mathsf{new}();$$
$$y = \mathsf{new}();$$
$$\boxed{x[A]} = y;$$
$$z = \boxed{y};$$
$$\mathsf{ret} = \boxed{z};$$

Accessible from the outside world are memory blocks which:

- are assigned to a global variable such as ret; or
- are reachable from global variables.

## ... in the Example:

$$x = \mathsf{new}();$$
$$y = \boxed{\mathsf{new}()};$$
$$\boxed{x[A]} = y;$$
$$z = \boxed{y};$$
$$\mathsf{ret} = \boxed{z};$$

## We conclude:

- The objects which have been allocated by the first `new()` may never escape.
- They can be allocated on the stack :-)

## Warning:

This is only meaningful if only few such objects are allocated during a method call :-(

If a local `new()` occurs within a loop, we still may allocate the objects in the heap ;-)

## Extension: Procedures

- We require an interprocedural points-to analysis :-)
- We know the whole program, we can, e.g., merge the control-flow graphs of all procedures into one and compute the points-to information for this.
- Warning: If we always use the same global variables $y_1, y_2, \ldots$ for (the simulation of) parameter passing, the computed information is necessarily imprecise :-(
- If the whole program is not known, we must assume that each reference which is known to a procedure escapes :-((

## 3.4 Wrap-Up

We have considered various optimizations for improving hardware utilization.

### Arrangement of the Optimizations:

- First, global restructuring of procedures/functions and of loops for better memory behavior ;-)
- Then local restructuring for better utilization of the instruction set and the processor parallelism :-)
- Then register allocation and finally,
- Peephole optimization for the final kick ...

| | |
|---|---|
| Procedures: | Tail Recursion + Inlining<br>Stack Allocation |
| Loops: | Iteration Reordering<br>$\longrightarrow$ if-Distribution<br>$\longrightarrow$ for-Distribution<br>Value Caching |
| Bodies: | Life-Range Splitting (SSA)<br>Instruction Selection<br>Instruction Scheduling with<br>$\longrightarrow$ Loop Unrolling<br>$\longrightarrow$ Loop Fusion |
| Instructions: | Register Allocation<br>Peephole Optimization |