The Effects of Edges:

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f\,(y_1 \cup y_2) & = & (u \in y_1 \cup y_2)\, ?\, b : \emptyset \\
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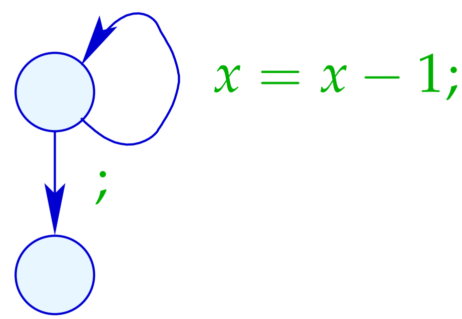
\end{aligned}
$$

$\Longrightarrow$ the constraint system yields the MOP :-))

- True liveness detects more superfluous assignments than repeated liveness !!!



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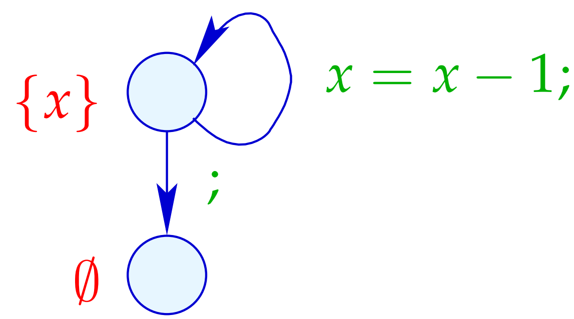
Liveness:



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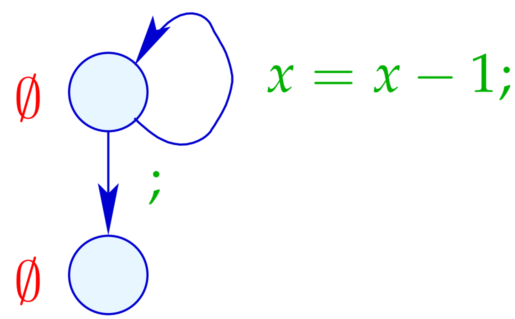
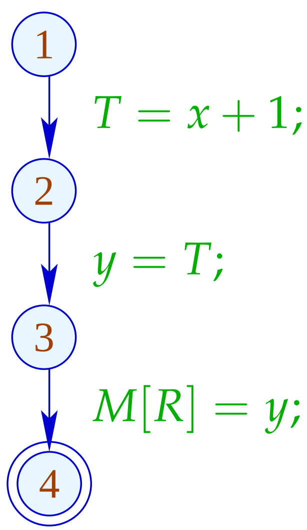
True Liveness:



1.3 Removing Superfluous Moves

Example:



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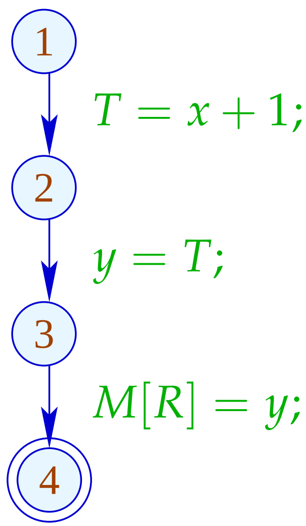

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Instead of y, we could also store T :-)

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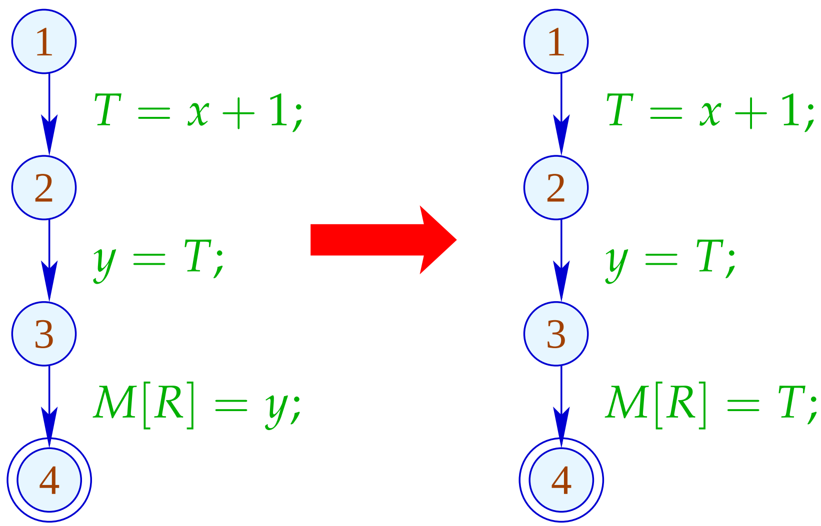

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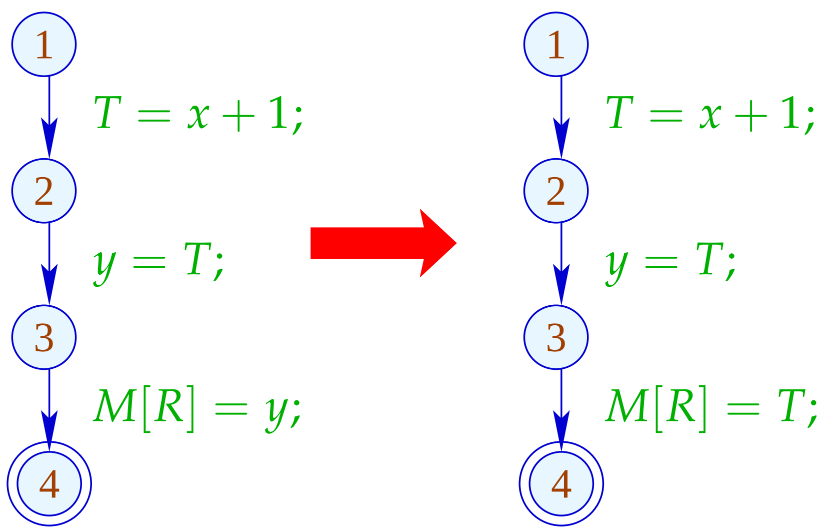

Advantage: Now, y has become dead :-))

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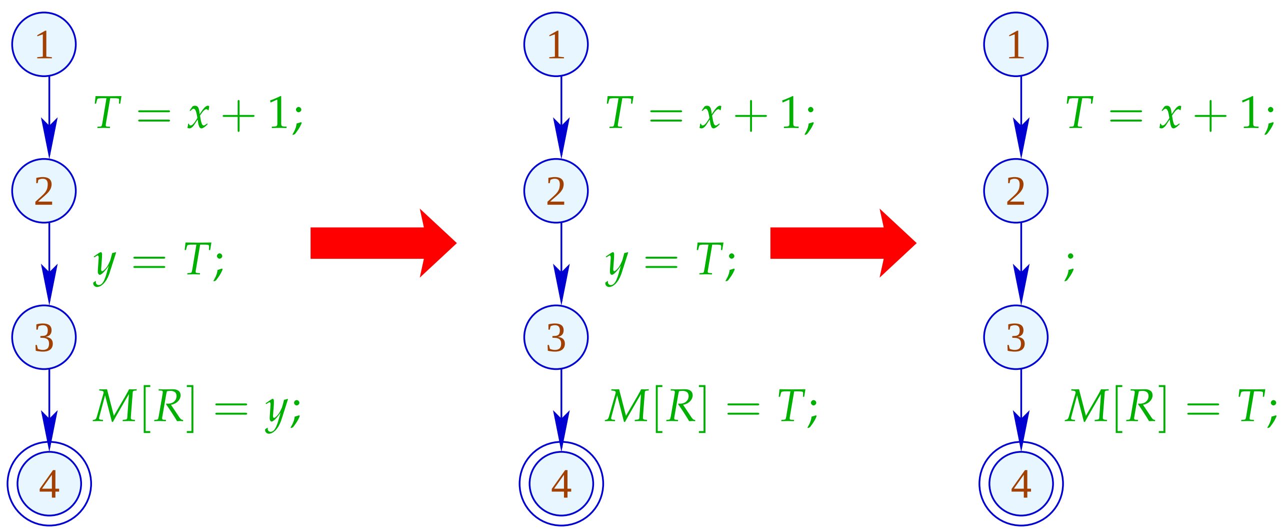

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Idea:

For each expression, we record the variable which currently contains its value :-)

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For each expression, we record the variable which currently contains its value :-)

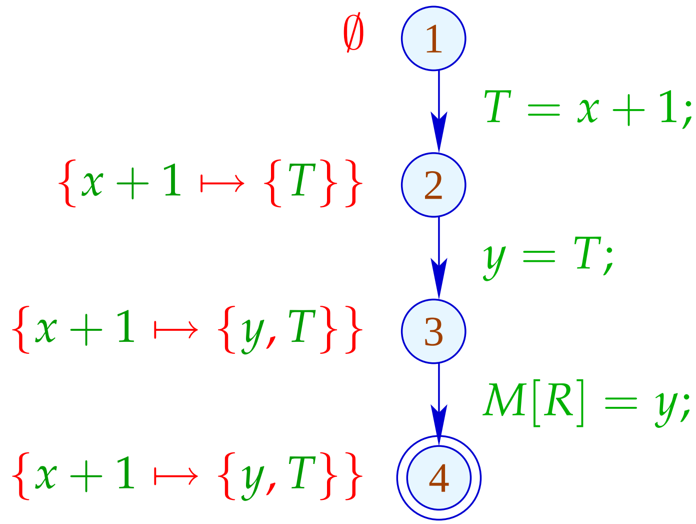
We use: $\mathbb{V} = Expr \to 2^{Vars}$ and define:

$$
\begin{array}{lcl}
[\![;]\!]^\sharp\, V & = & V \\
[\![\mathrm{Pos}(e)]\!]^\sharp\, V\, e' & = & [\![\mathrm{Neg}(e)]\!]^\sharp\, V\, e' \;=\; \begin{cases} \emptyset & \text{if } e' = e \\ V\, e' & \text{otherwise} \end{cases} \\
\ldots & &
\end{array}
$$

$$
\begin{array}{lcl}
[\![x = c;]\!]^\sharp\, V\, e' & = & \begin{cases} (V\, c) \cup \{x\} & \text{if} \quad e' = c \\ (V\, e')\backslash\{x\} & \text{otherwise} \end{cases} \\
[\![x = y;]\!]^\sharp\, V\, e & = & \begin{cases} (V\, e) \cup \{x\} & \text{if} \quad y \in V\, e \\ (V\, e)\backslash\{x\} & \text{otherwise} \end{cases} \\
[\![x = e;]\!]^\sharp\, V\, e' & = & \begin{cases} \{x\} & \text{if} \quad e' = e \\ (V\, e')\backslash\{x\} & \text{otherwise} \end{cases} \\
[\![x = M[c];]\!]^\sharp\, V\, e' & = & (V\, e')\backslash\{x\} \\
[\![x = M[y];]\!]^\sharp\, V\, e' & = & (V\, e')\backslash\{x\} \\
[\![x = M[e];]\!]^\sharp\, V\, e' & = & \begin{cases} \emptyset & \text{if } e' = e \\ (V\, e')\backslash\{x\} & \text{otherwise} \end{cases}
\end{array}
$$

$//$ analogously for the diverse stores

In the Example:



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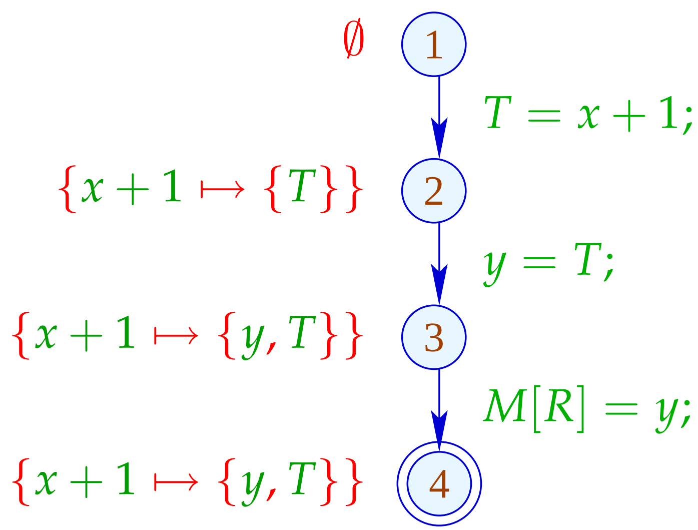


$\longrightarrow$ We propagate information in forward direction :-)

At *start* , $V_0\, e = \emptyset$ for all e;

$\longrightarrow$ $\sqsubseteq \subseteq \mathbb{V} \times \mathbb{V}$ is defined by:

$$V_1 \sqsubseteq V_2 \quad \text{iff} \quad V_1\, e \supseteq V_2\, e \quad \text{for all} \quad e$$

Observation:

The new effects of edges are distributive:

To show this, we consider the functions:

(1) $\quad f_1^x \, V \, e = (V \, e) \backslash \{x\}$

(2) $\quad f_2^{e,a} \, V = V \oplus \{e \mapsto a\}\}$

(3) $\quad f_3^{x,y} \, V \, e = (y \in V \, e) \, ? \, (V \, e \cup \{x\}) : ((V \, e) \backslash \{x\})$

Obviously, we have:

$$\begin{array}{lll} [\![x = e;]\!]^\sharp & = & f_2^{e,\{x\}} \circ f_1^x \\ [\![x = y;]\!]^\sharp & = & f_3^{x,y} \\ [\![x = M[e];]\!]^\sharp & = & f_2^{e,\emptyset} \circ f_1^x \end{array}$$

By closure under composition, the assertion follows :-))

(1) For $f\,V\,e = (V\,e)\backslash\{x\}$, we have:

$$
\begin{aligned}
f\,(V_1 \sqcup V_2)\,e &= ((V_1 \sqcup V_2)\,e)\backslash\{x\} \\
&= ((V_1\,e) \cap (V_2\,e))\backslash\{x\} \\
&= ((V_1\,e)\backslash\{x\}) \cap ((V_2\,e)\backslash\{x\}) \\
&= (f\,V_1\,e) \cap (f\,V_2\,e) \\
&= (f\,V_1 \sqcup f\,V_2)\,e \qquad \text{:-)}
\end{aligned}
$$

(2) For $f\,V = V \oplus \{e \mapsto a\}$, we have:

$$
\begin{array}{lll}
f\,(V_1 \sqcup V_2)\,e' & = & ((V_1 \sqcup V_2) \oplus \{e \mapsto a\})\,e' \\
& = & (V_1 \sqcup V_2)\,e' \\
& = & (f\,V_1 \sqcup f\,V_2)\,e' \qquad \text{given that} \quad e \neq e'
\end{array}
$$

$$
\begin{array}{lll}
f\,(V_1 \sqcup V_2)\,e & = & ((V_1 \sqcup V_2) \oplus \{e \mapsto a\})\,e \\
& = & a \\
& = & ((V_1 \oplus \{e \mapsto a\})\,e) \sqcap ((V_2 \oplus \{e \mapsto a\})\,e) \\
& = & (f\,V_1 \sqcup f\,V_2)\,e \qquad \text{:-)}
\end{array}
$$

(3) For $f\,V\,e = (y \in V\,e)\;?\;(V\,e \cup \{x\}) : ((V\,e)\backslash\{x\})$, we have:

$$
\begin{array}{lll}
f\,(V_1 \sqcup V_2)\,e & = & (((V_1 \sqcup V_2)\,e)\backslash\{x\}) \cup (y \in (V_1 \sqcup V_2)\,e)\;?\;\{x\} : \emptyset \\
& = & ((V_1\,e \cap V_2\,e)\backslash\{x\}) \cup (y \in (V_1\,e \cap V_2\,e))\;?\;\{x\} : \emptyset \\
& = & ((V_1\,e \cap V_2\,e)\backslash\{x\}) \cup \\
& & ((y \in V_1\,e)\;?\;\{x\} : \emptyset) \cap ((y \in V_2\,e)\;?\;\{x\} : \emptyset) \\
& = & (((V_1\,e)\backslash\{x\}) \cup (y \in V_1\,e)\;?\;\{x\} : \emptyset) \cap \\
& & (((V_2\,e)\backslash\{x\}) \cup (y \in V_2\,e)\;?\;\{x\} : \emptyset) \\
& = & (f\,V_1 \sqcup f\,V_2)\,e \qquad\qquad :\text{-})
\end{array}
$$

We conclude:

→ Solving the constraint system returns the MOP solution :-)

→ Let $\mathcal{V}$ denote this solution.

If $x \in \mathcal{V}[u]\, e$, then x at u contains the value of e — which we have stored in T_e

$\Longrightarrow$

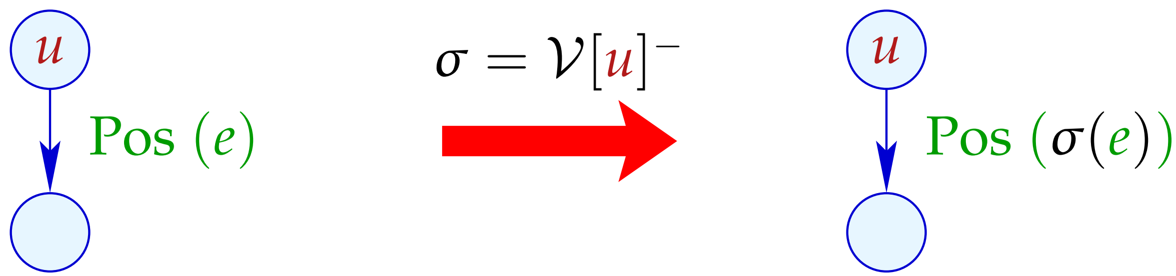
the access to x can be replaced by the access to T_e :-)

For $V \in \mathbb{V}$, let V^- denote the variable substitution with:

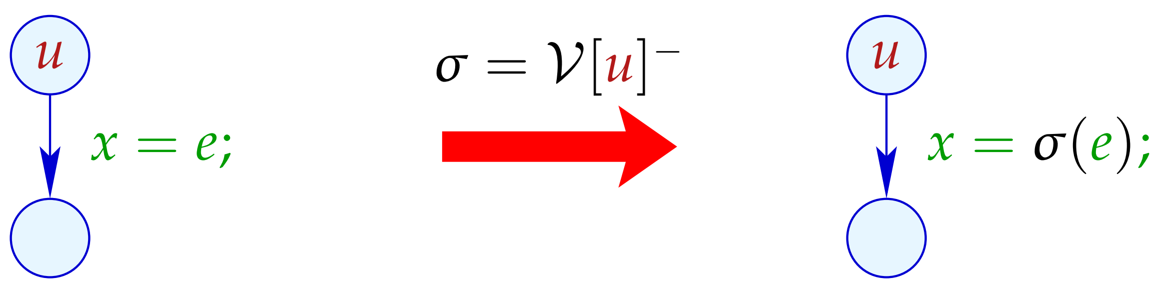
$$V^- x = \begin{cases} T_e & \text{if } x \in V\, e \\ x & \text{otherwise} \end{cases}$$

if $V\, e \cap V\, e' = \emptyset$ for $e \neq e'$. Otherwise: $V^- x = x$:-)

Transformation 3:



... analogously for edges with Neg (e)



Transformation 3 (cont.):

$$\begin{tikzcd} u \arrow[d, "x = M[e];"] \\ {} \end{tikzcd} \quad \xRightarrow{\sigma = \mathcal{V}[u]^-} \quad \begin{tikzcd} u \arrow[d, "x = M[\sigma(e)];"] \\ {} \end{tikzcd}$$

$$\begin{tikzcd} u \arrow[d, "M[e_1] = e_2;"] \\ {} \end{tikzcd} \quad \xRightarrow{\sigma = \mathcal{V}[u]^-} \quad \begin{tikzcd} u \arrow[d, "M[\sigma(e_1)] = \sigma(e_2);"] \\ {} \end{tikzcd}$$

Procedure as a whole:

(1) Availability of expressions: T1

- + removes arithmetic operations
- − inserts superfluous moves

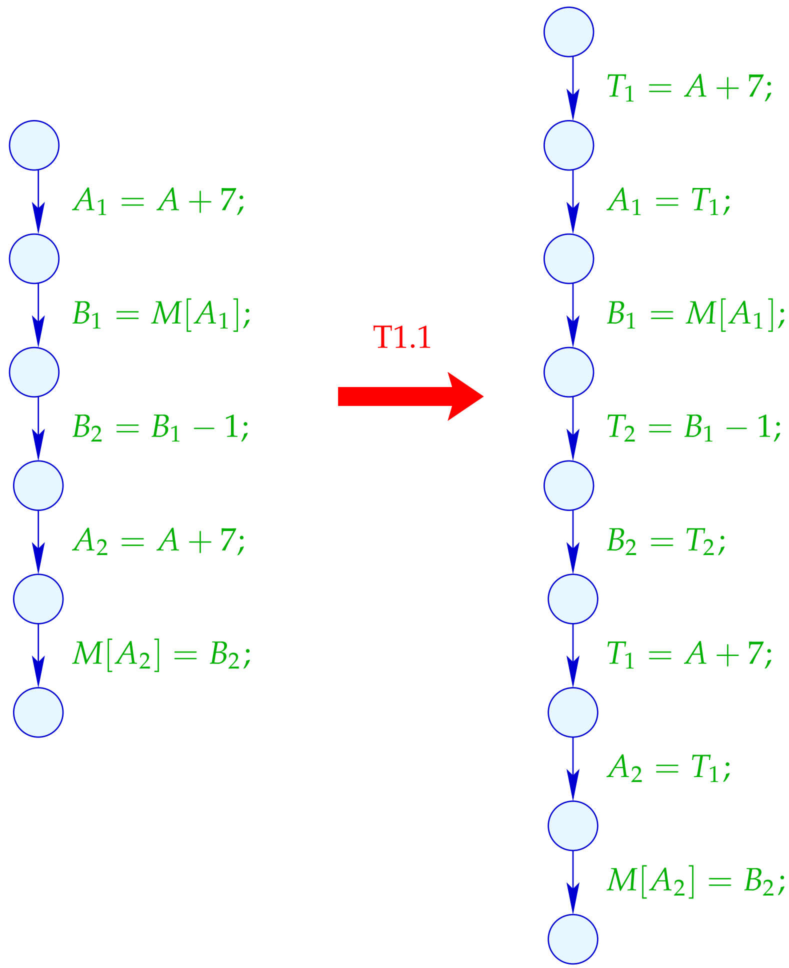
(2) Values of variables: T3

- + creates dead variables

(3) (true) liveness of variables: T2

- + removes assignments to dead variables

Example: `a[7]--;`


$A_1 = A + 7;$
$B_1 = M[A_1];$
$B_2 = B_1 - 1;$
$A_2 = A + 7;$
$M[A_2] = B_2;$
T1.1
$T_1 = A + 7;$
$A_1 = T_1;$
$B_1 = M[A_1];$
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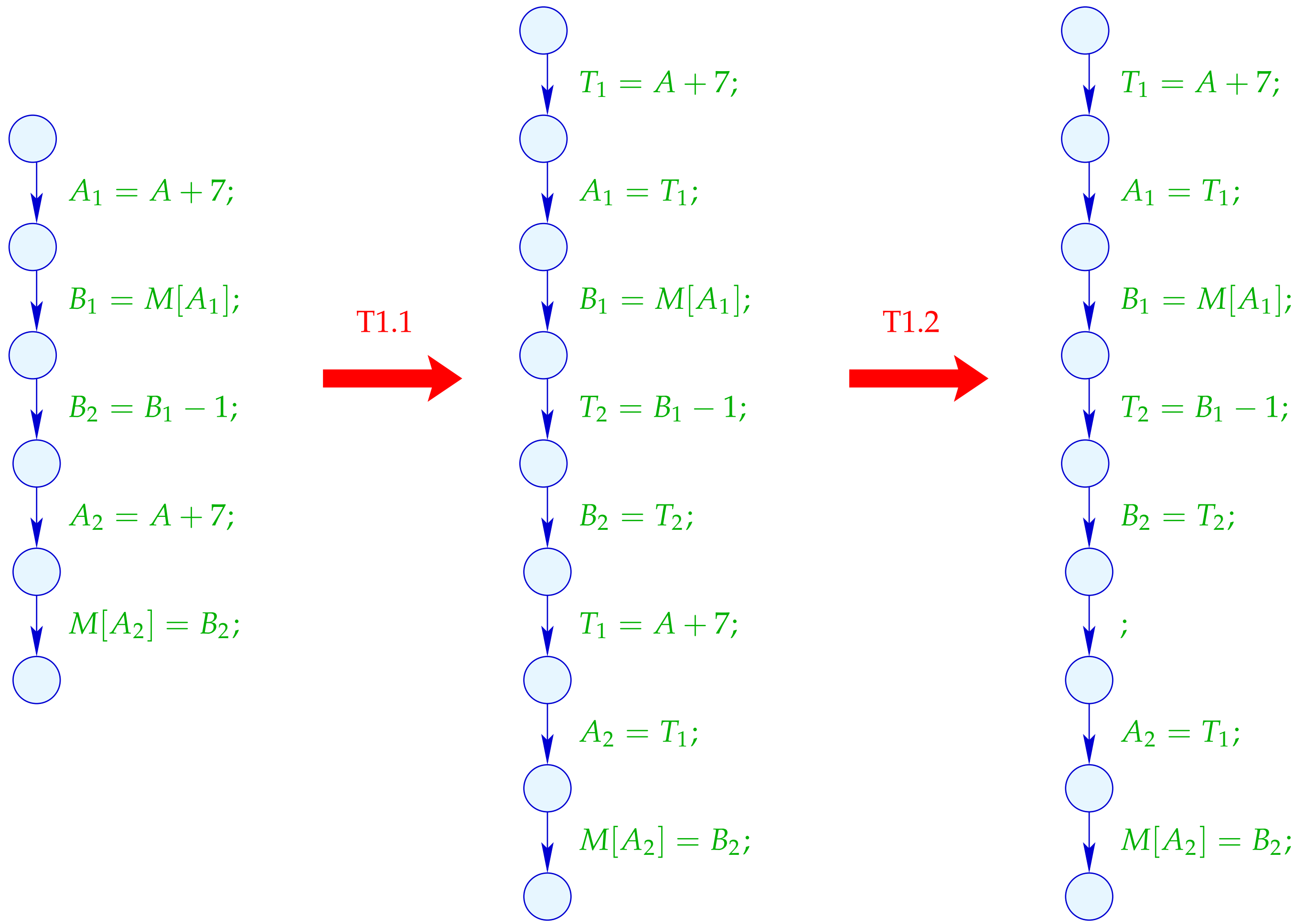
$A_1 = A + 7;$
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T1.2
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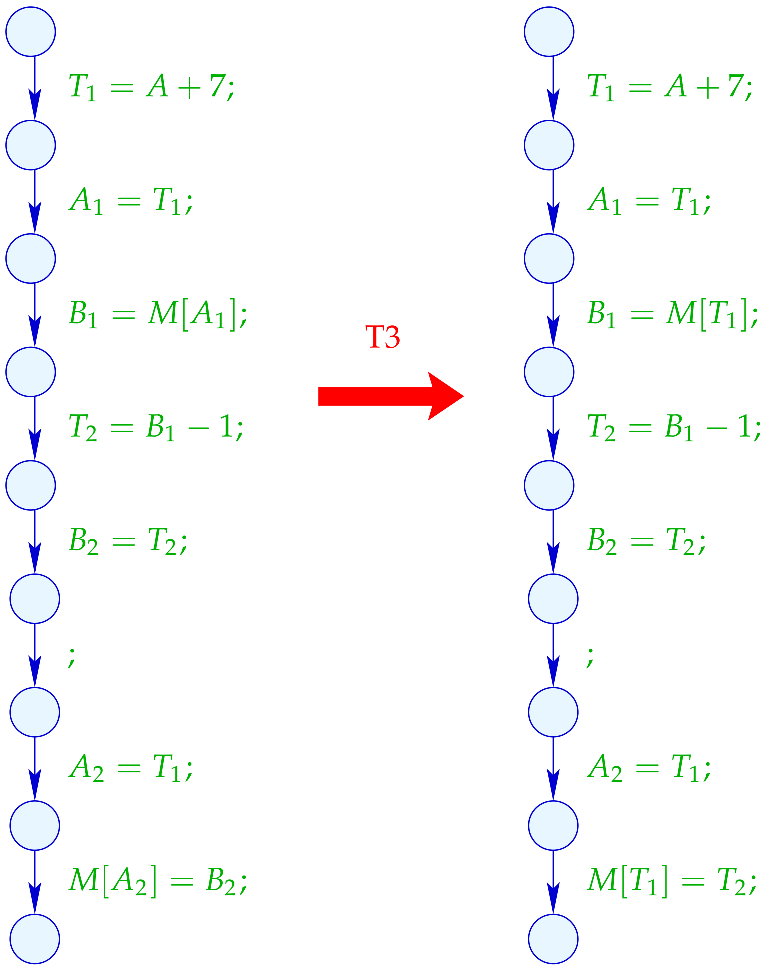
$A_1 = T_1;$
$B_1 = M[A_1];$
$T_2 = B_1 - 1;$
$B_2 = T_2;$
$;$
$A_2 = T_1;$
$M[A_2] = B_2;$

Example (cont.): `a[7]--;`



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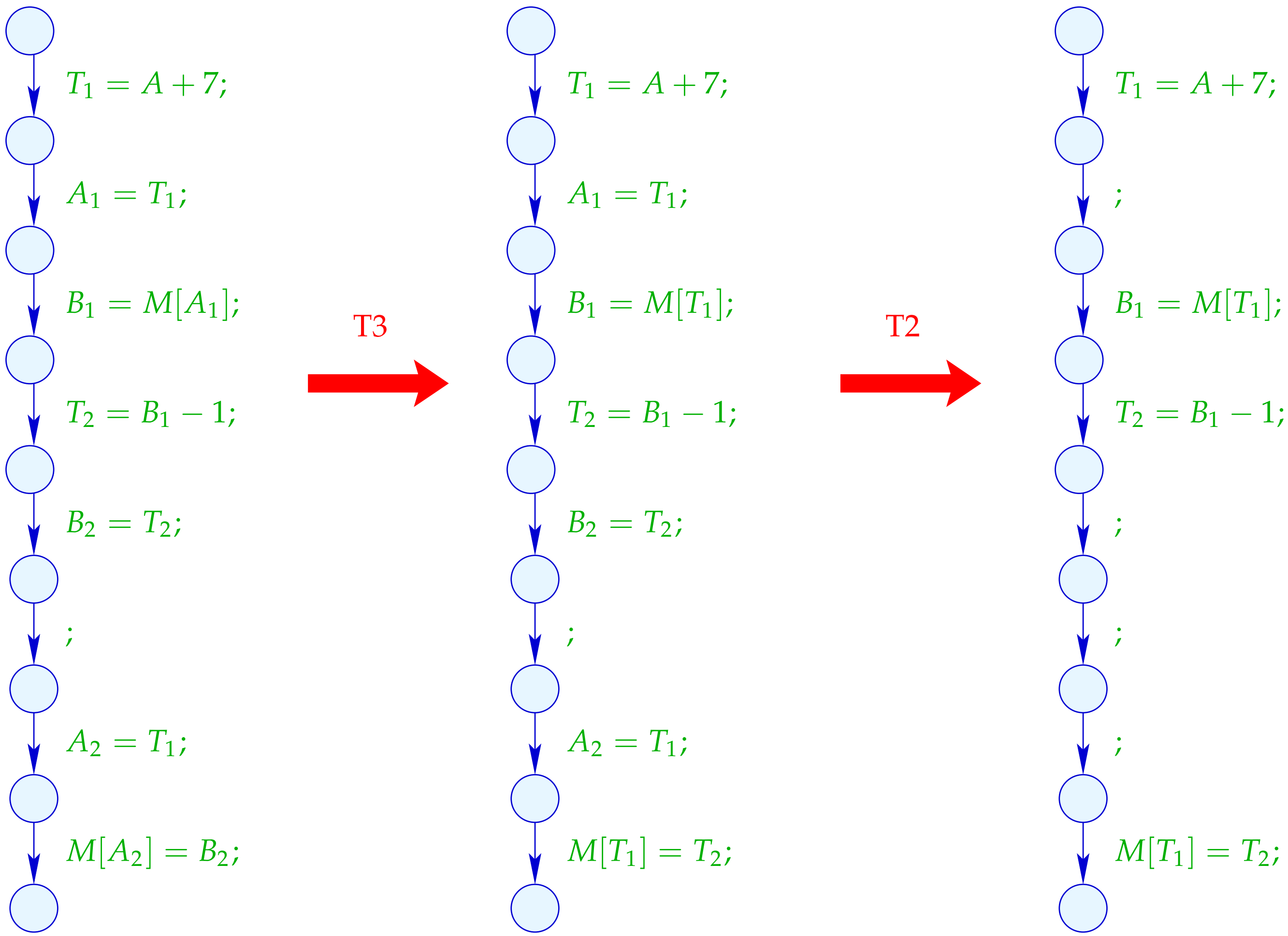


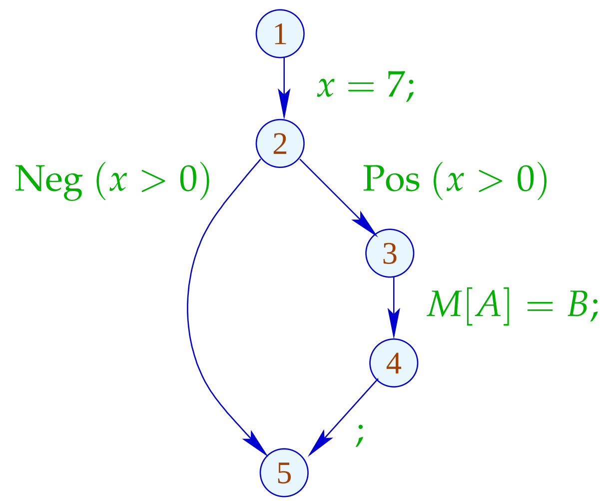
1.4 Constant Propagation

Idea:

Execute as much of the code at compile-time as possible!

Example:

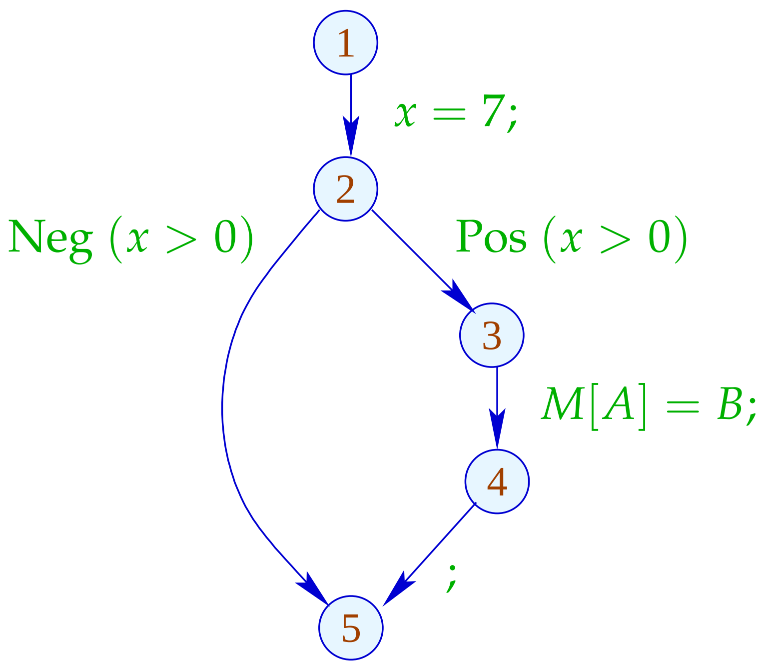
$$x = 7;$$
$$\text{if } (x > 0)$$
$$\quad M[A] = B;$$



Obviously, x has always the value 7 :-)

Thus, the memory access is always executed :-))

Goal:



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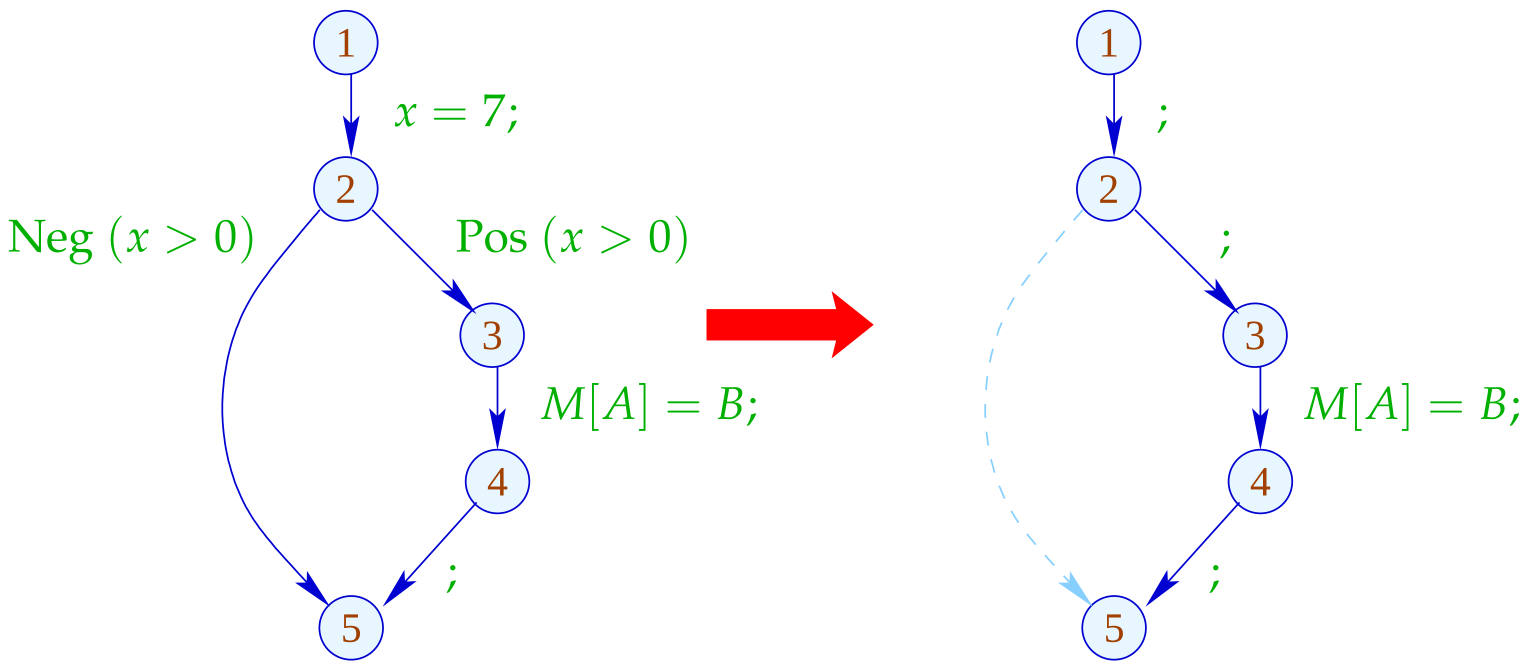

Generalization: Partial Evaluation

Neil D. Jones, DIKU, Kopenhagen