## Approach:

Identify

- . . . loops;
- . . . iteration variables;
- . . . constants;
- . . . the matching use structures.

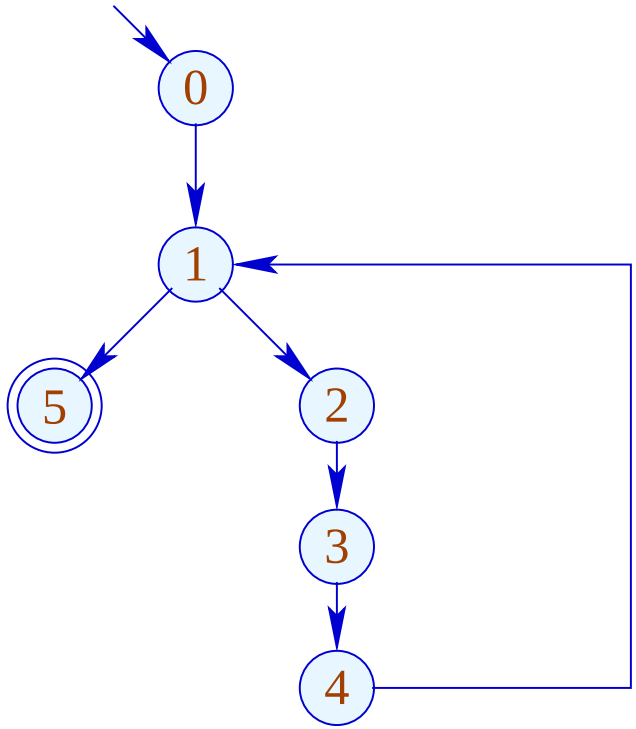
## Loops:

... are identified through the node $v$ with back edge $(\_, \_, v)$ :-)

For the sub-graph $G_v$ of the cfg on $\{w \mid v \Rightarrow w\}$, we define:

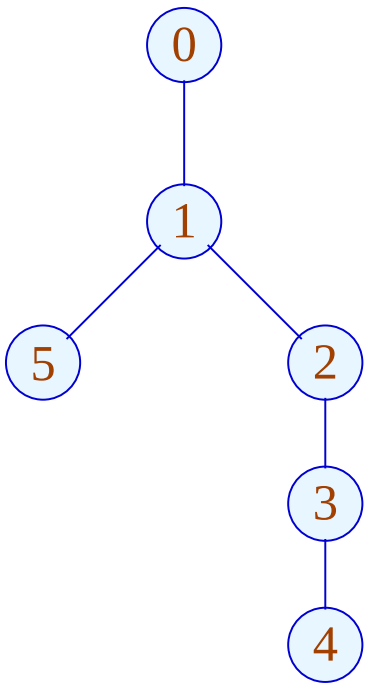
$$\mathsf{Loop}[v] \;=\; \{w \mid w \rightarrow^* v \;\text{ in }\; G_v\}$$

## Example:




| | $\mathcal{P}$ |
|---|---|
| 0 | $\{0\}$ |
| 1 | $\{0, 1\}$ |
| 2 | $\{0, 1, 2\}$ |
| 3 | $\{0, 1, 2, 3\}$ |
| 4 | $\{0, 1, 2, 3, 4\}$ |
| 5 | $\{0, 1, 5\}$ |

## Example:

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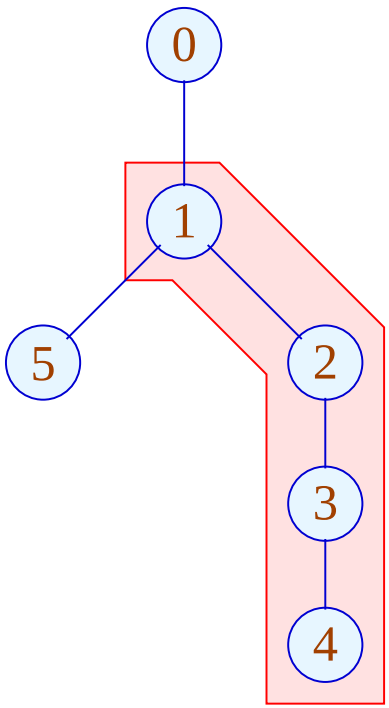


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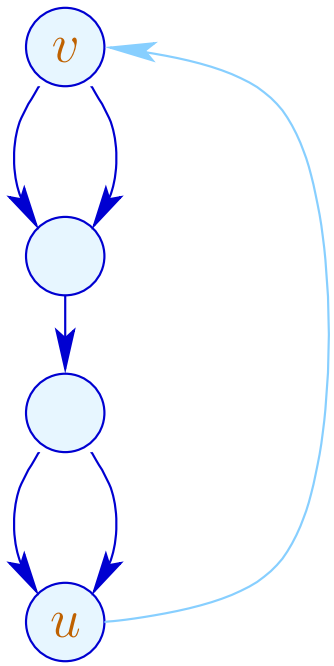
We are interested in edges which during each iteration are executed exactly once:




This property can be expressed by means of the pre-dominator relation ...

Assume that $(u, \_, v)$ is the back edge.

Then edges $k = (u_1, \_, v_1)$ could be selected such that:

- $v$ pre-dominates $u_1$;
- $u_1$ pre-dominates $v_1$;
- $v_1$ predominates $u$.

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On the level of source programs, this is trivial:

$$\begin{array}{l} \mathsf{do}\ \{\ s_1 \ldots s_k \\ \}\ \mathsf{while}\ (e); \end{array}$$

The desired assignments must be among the $s_i$ :-)

## Iteration Variable:

$i$ is an iteration variable if the only definition of $i$ inside the loop occurs at an edge which separates the body and is of the form:

$$i = i + h;$$

for some loop constant $h$ .

A loop constant is simply a constant (e.g., 42), or slightly more libaral, an expression which only depends on variables which are not modified during the loop :-)

# (3) Differences for Sets

Consider the fixpoint computation:

$$
\begin{array}{l}
x = \emptyset; \\
\mathsf{for}\ \ (t = F\,x; t \not\subseteq x; \boxed{t = F\,x;}) \\
\qquad x = x \cup t;
\end{array}
$$

If $F$ is distributive, it could be replaced by:

$$
\begin{array}{l}
x = \emptyset; \\
\mathsf{for}\ \ (\Delta = F\,x; \Delta \neq \emptyset; \boxed{\Delta = (F\,\Delta) \setminus x;}) \\
\qquad x = x \cup \Delta;
\end{array}
$$

The function $F$ must only be computed for the smaller sets $\Delta$ :-)

semi-naive iteration

Instead of the sequence: $\emptyset \subseteq F(\emptyset) \subseteq F^2(\emptyset) \subseteq \ldots$

we compute: $\Delta_1 \cup \Delta_2 \cup \ldots$

where:
$$\begin{aligned} \Delta_{i+1} &= F(F^i(\emptyset)) \backslash F^i(\emptyset) \\ &= F(\Delta_i) \backslash (\Delta_1 \cup \ldots \cup \Delta_i) \quad \text{with } \Delta_0 = \emptyset \end{aligned}$$

Assume that the costs of $F\ x$ is $1 + \#x$ .

Then the costs may sum up to:

| | |
|---|---|
| naive | $1 + 2 + \ldots + n + n = \frac{1}{2}n(n+3)$ |
| semi-naive | $2n$ |

where $n$ is the cardinality of the result.

$\Longrightarrow$ A linear factor is saved :-)

## 2.2 Peephole Optimization

Idea:

- Slide a small window over the program.
- Optimize agressively inside the window, i.e.,
  - $\rightarrow$ Eliminate redundancies!
  - $\rightarrow$ Replace expensive operations inside the window by cheaper ones!

## Examples:

$y = M[x]; x = x + 1; \quad \Longrightarrow \quad y = M[x{+}{+}];$

// given that there is a specific post-increment instruction :-)

$z = y - a + a; \quad \Longrightarrow \quad z = y;$

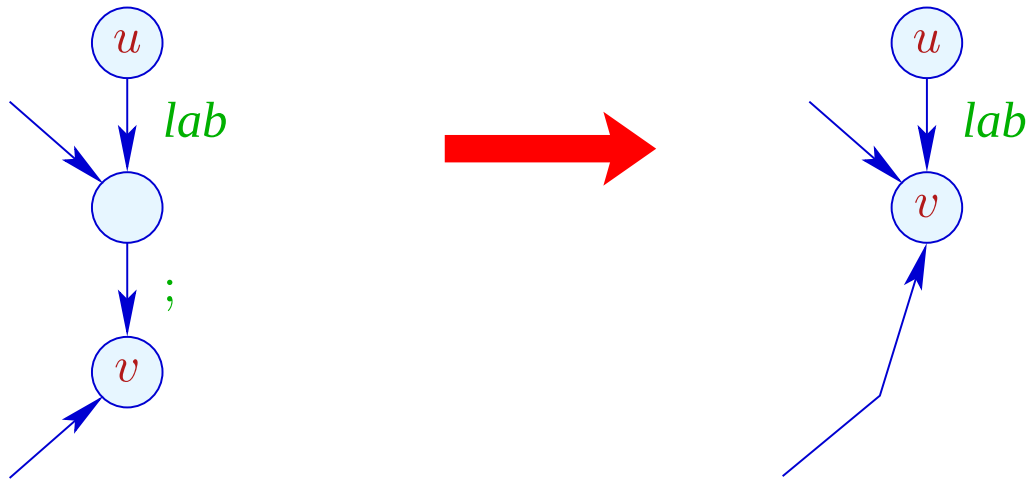
// algebraic simplifications :-)

$x = x; \quad \Longrightarrow \quad ;$

$x = 0; \quad \Longrightarrow \quad x = x \oplus x;$

$x = 2 \cdot x; \quad \Longrightarrow \quad x = x + x;$

## Important Subproblem: *nop*-Optimization




→ If $(v_1, ;, v)$ is an edge, $v_1$ has no further out-going edge.

→ Consequently, we can identify $v_1$ and $v$ :-)

→ The ordering of the identifications does not matter :-))

# Implementation:

- We construct a function $\mathsf{next} : Nodes \to Nodes$ with:

$$\mathsf{next}\ u = \begin{cases} \mathsf{next}\ v & \text{if} \quad (u, ;, v) \quad \text{edge} \\ u & \text{otherwise} \end{cases}$$

  Warning: This definition is only recursive if there are ;-loops ???

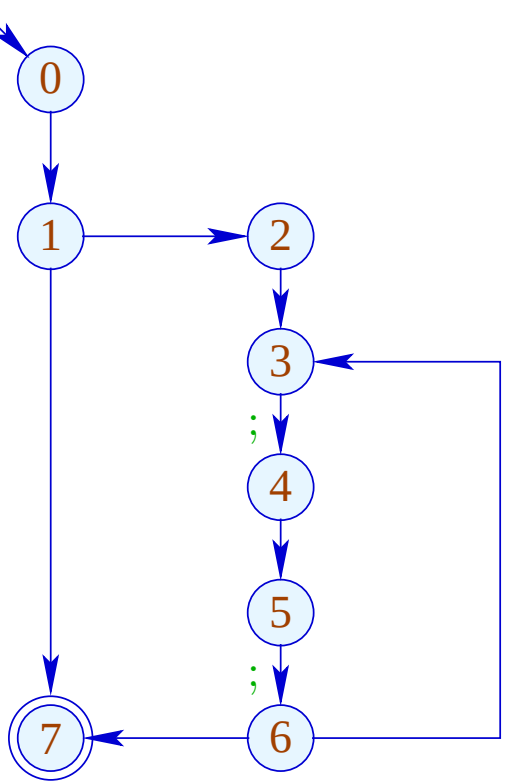
- We replace every edge:

$$(u, lab, v) \quad \Longrightarrow \quad (u, lab, \mathsf{next}\ v)$$

  ... whenever $lab \neq ;$

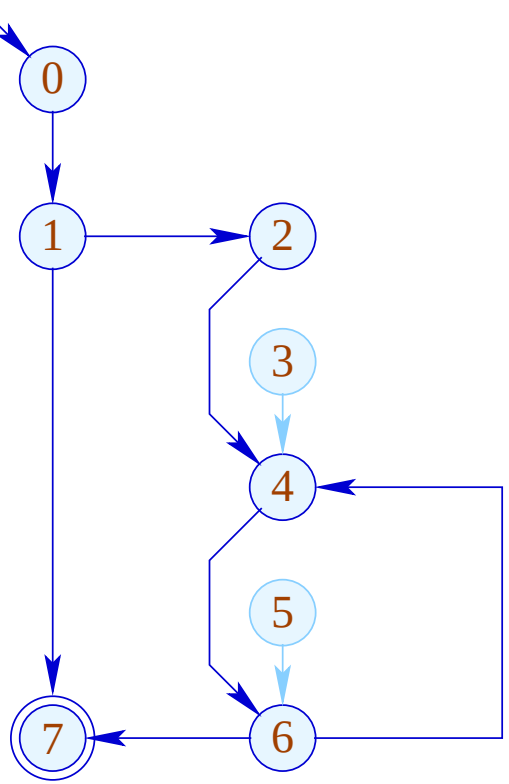
- All ;-edges are removed ;-)

# Example:




$$\begin{aligned} \text{next } 1 &= 1 \\ \text{next } 3 &= 4 \\ \text{next } 5 &= 6 \end{aligned}$$

# Example:

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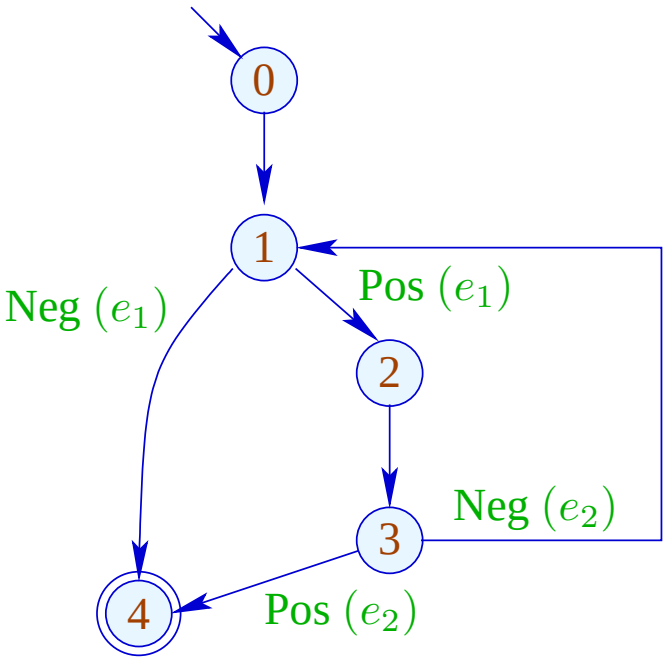
## 2. Subproblem: Linearization

After optimization, the CFG must again be brought into a linearly arrangement of instructions :-)

## Warning:

Not every linearization is equally efficient !!!

# Example:




0:

1: if $(e_1)$ goto 2;

4: halt

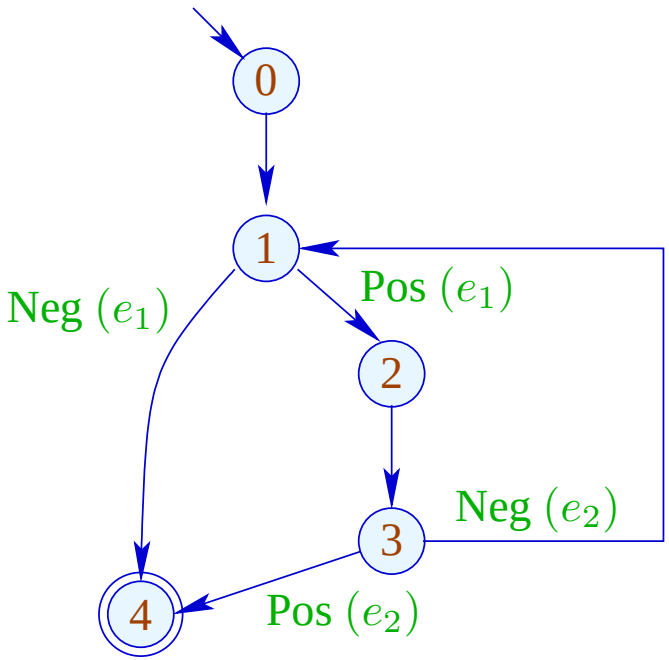
2: Rumpf

3: if $(e_2)$ goto 4;

goto 1;

Bad: The loop body is jumped into :-(

# Example:




0:

1: if $(!e_1)$ goto 4;

2: Rumpf

3: if $(!e_2)$ goto 1;

4: halt

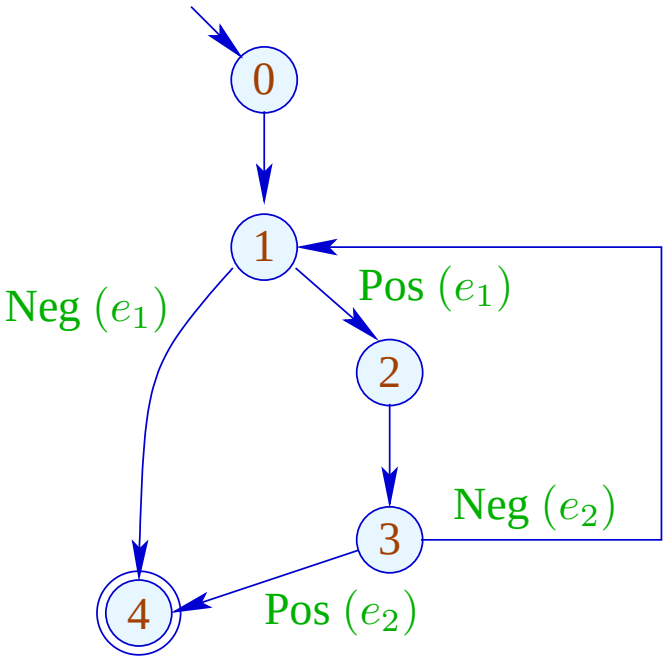
// better cache behavior :-)

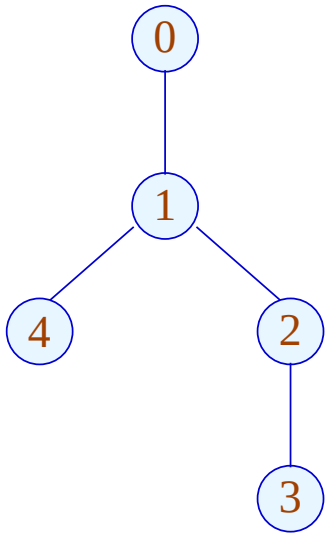
## Idea:

- Assign to each node a temperature!
- always jumps to
  - (1) nodes which have already been handled;
  - (2) colder nodes.
- Temperature $\approx$ nesting-depth

For the computation, we use the pre-dominator tree and strongly connected components ...

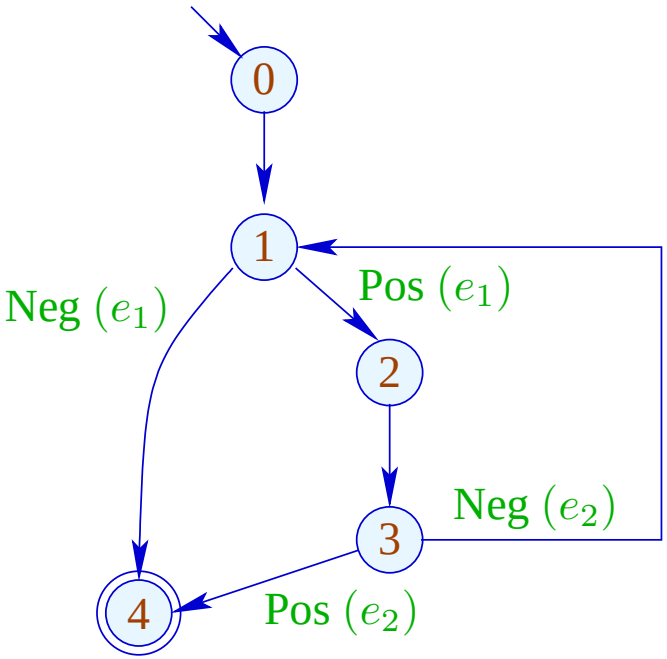
# ... in the Example:

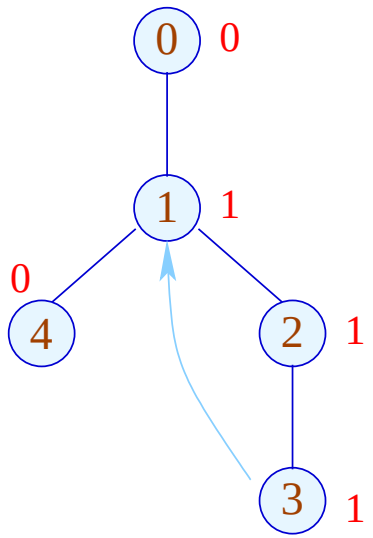


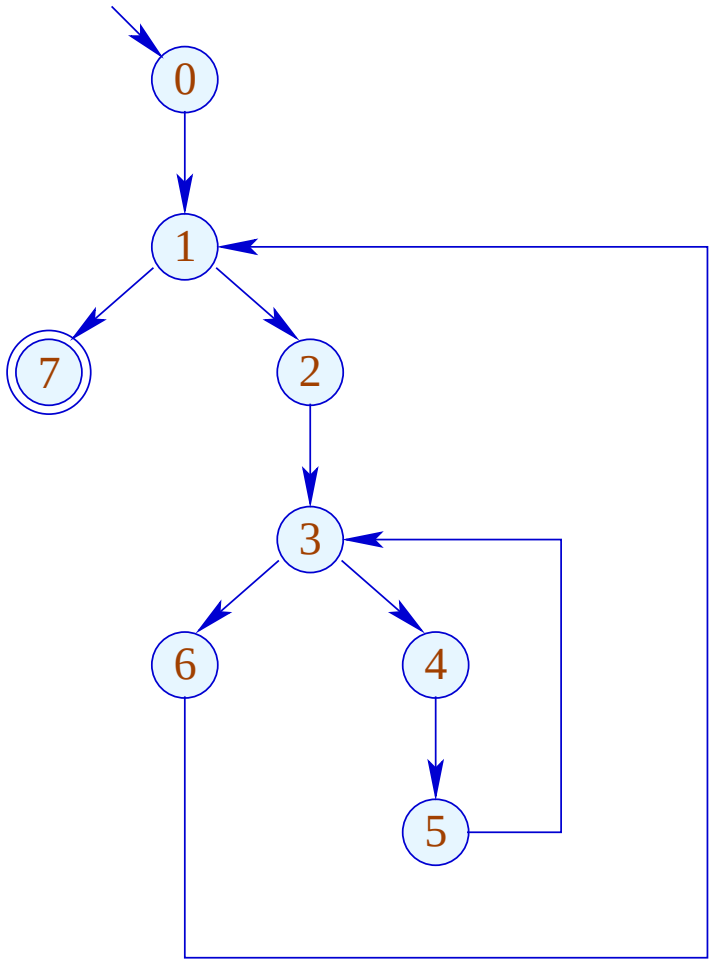

The sub-tree with back edge is hotter ...

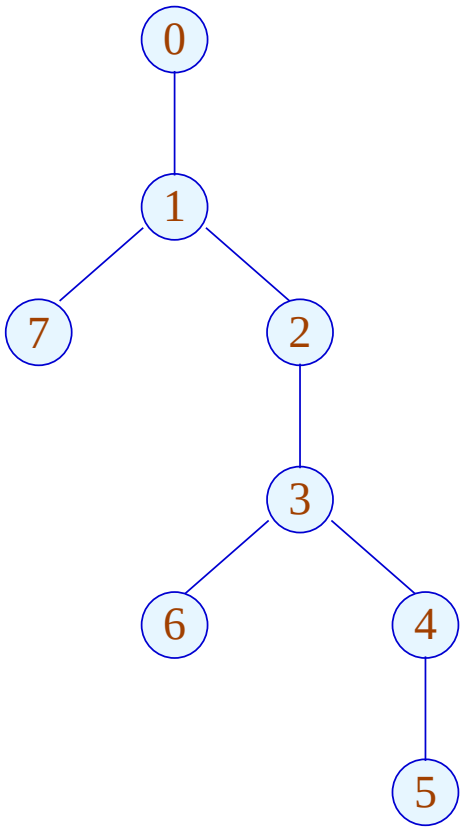
# ... in the Example:



# More Complicated Example:



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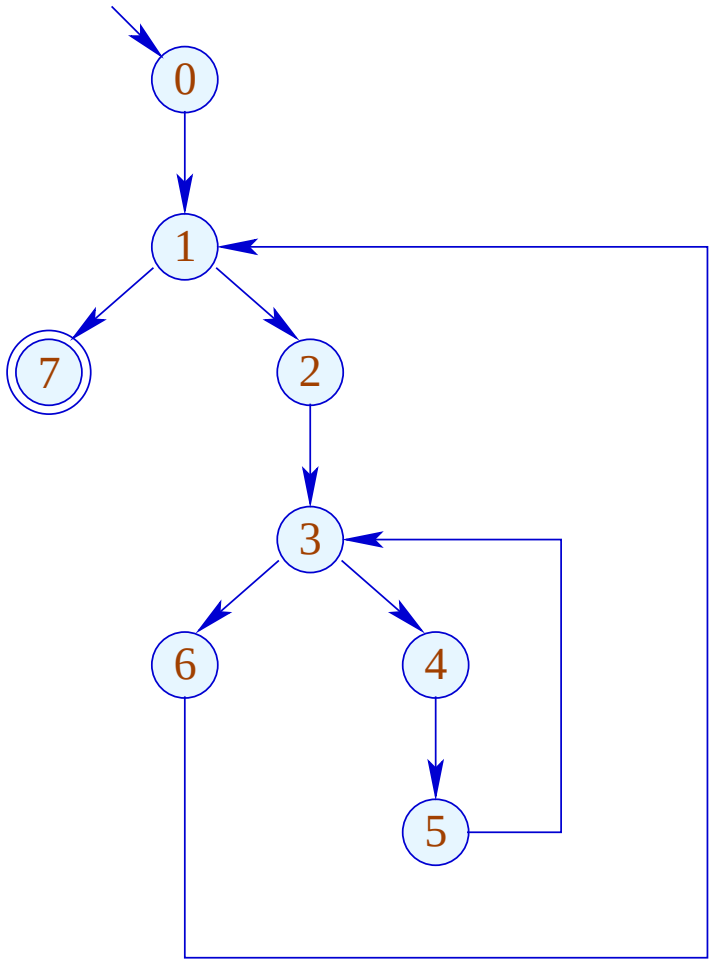


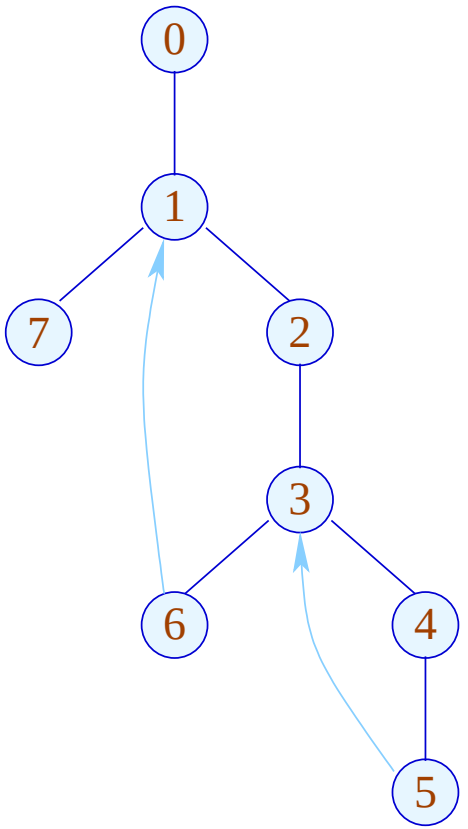

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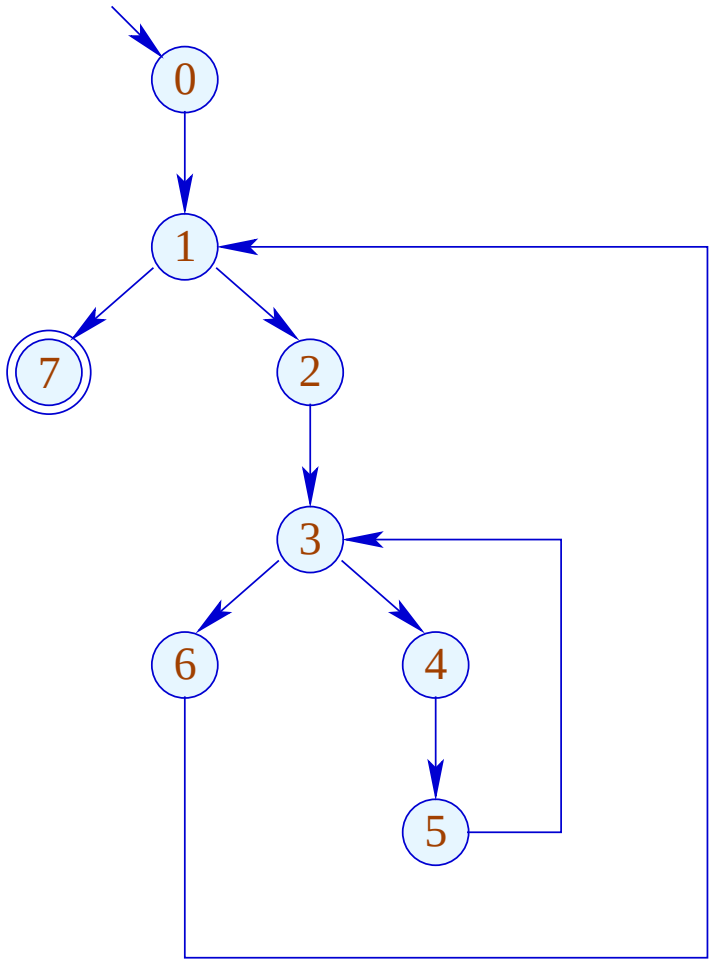
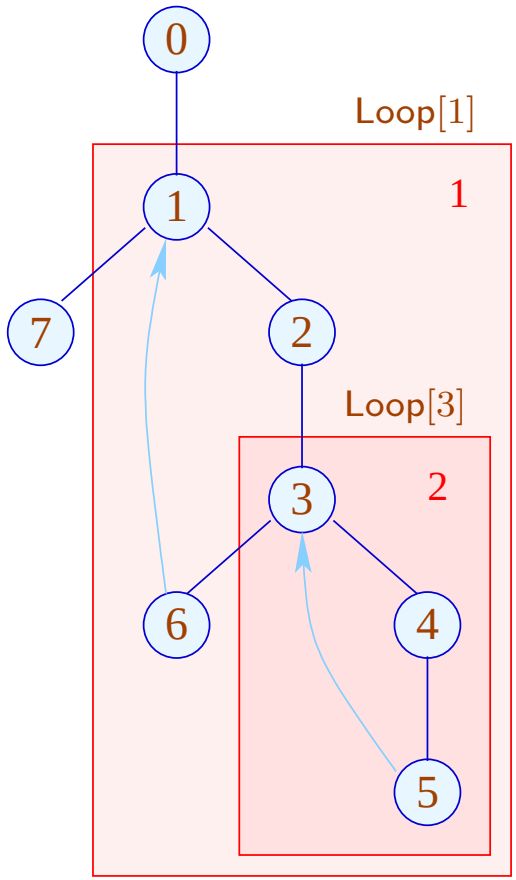
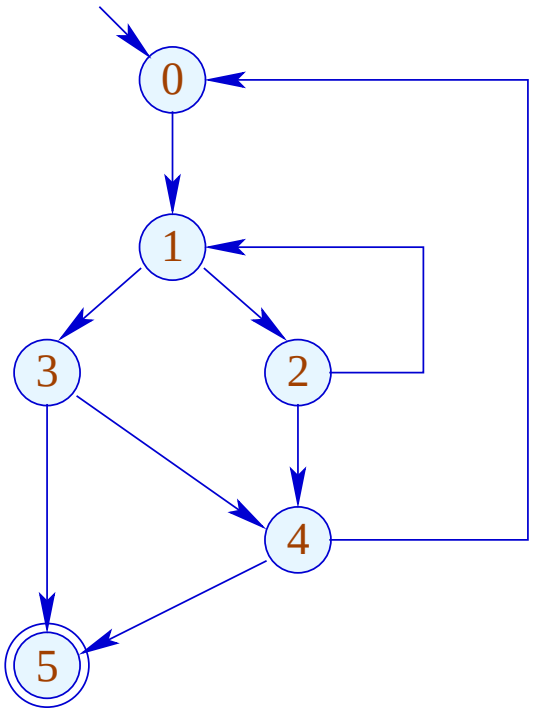
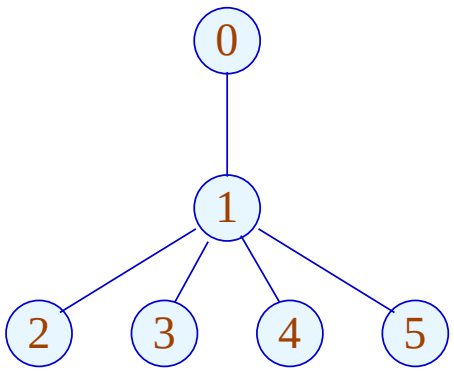


Our definition of Loop implies that (detected) loops are necessarily nested :-)

Is is also meaningful for do-while-loops with breaks ...



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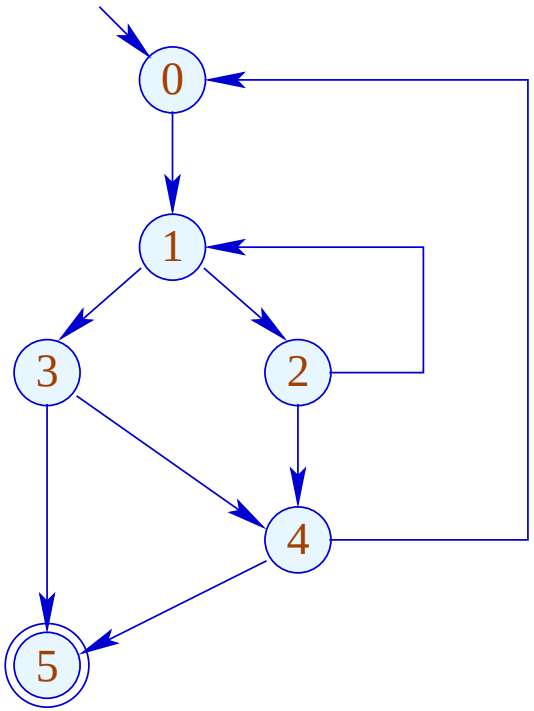


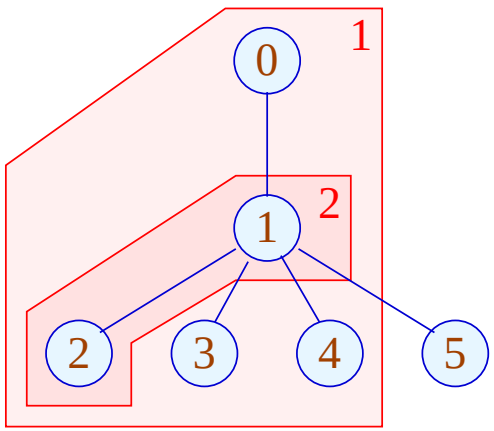

## Summary: The Approach

(1) For every node, determine a temperature;

(2) Pre-order-DFS over the CFG;

- → If an edge leads to a node we already have generated code for, then we insert a jump.
- → If a node has two successors with different temperature, then we insert a jump to the colder of the two.
- → If both successors are equally warm, then it does not matter ;-)

## 2.3 Procedures

We extend our mini-programming language by procedures without parameters and procedure calls.

For that, we introduce a new statement:

$$f\,();$$

Every procedure $f$ has a definition:

$$f\,()\ \{\ stmt^*\ \}$$

Additionally, we distinguish between global and local variables.

Program execution starts with the call of a procedure $\mathsf{main}\,()$ .

# Example:

```
int a, ret;
main () {
        a = 3;
        f();
        M[17] = ret;
        ret = 0;
}
```

```
f () {
        int b;
        if (a ≤ 1) {ret = 1; goto exit;}
        b = a;
        a = b − 1;
        f();
        ret = b · ret;
exit :
}
```

Such programs can be represented by a set of CFGs: one for each procedure ...

# ... in the Example: