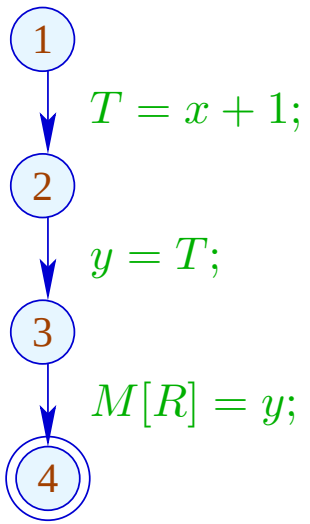
## 1.3 Removing Superfluous Moves

Example:

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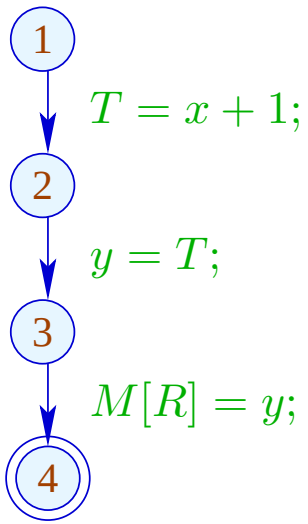


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Instead of $y$, we could also store $T$ :-)

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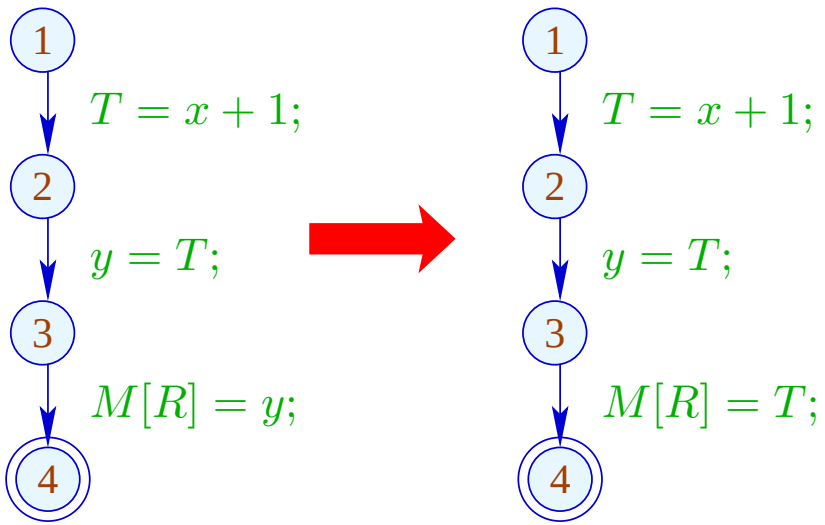


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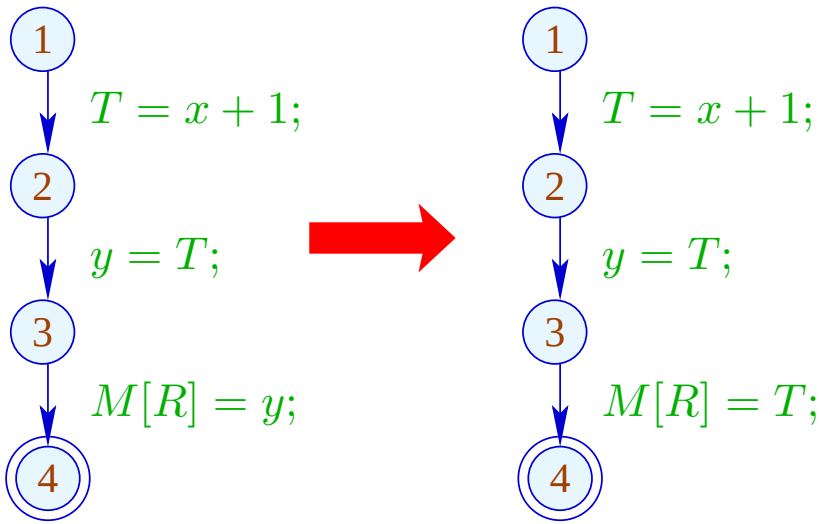


Advantage: Now, $y$ has become dead :-))

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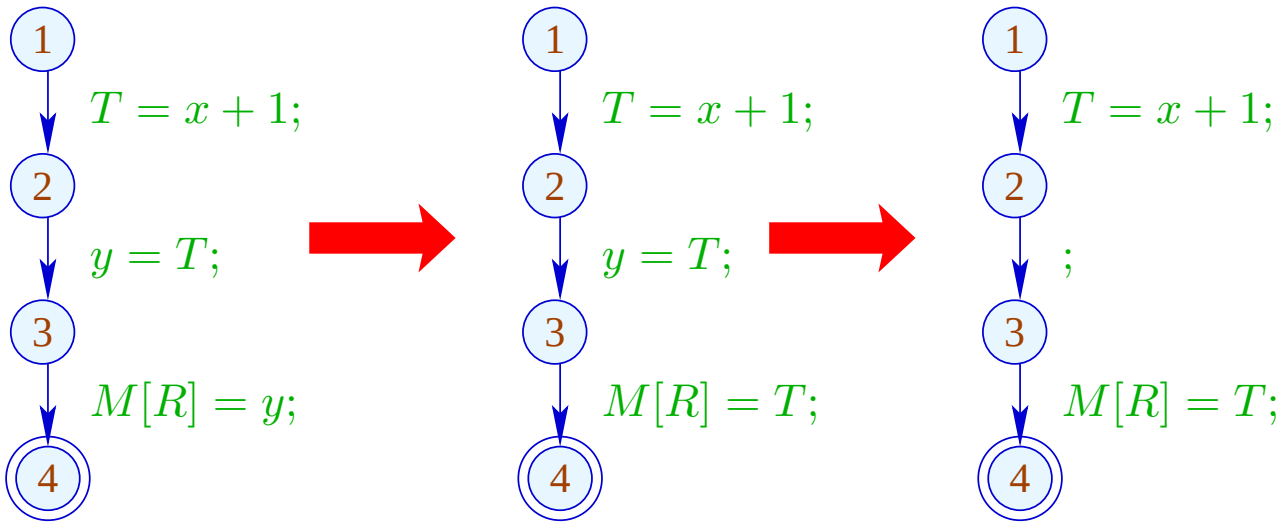


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For each expression, we record the variable which currently contains its value :-)

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## Idea:

For each expression, we record the variable which currently contains its value :-)

We use: $\mathbb{V} = \mathit{Expr} \to 2^{\mathit{Vars}}$ and define:

$$
\begin{array}{lcl}
[\![;]\!]^\sharp\, V & = & V \\
[\![\mathrm{Pos}(e)]\!]^\sharp\, V\, e' & = & [\![\mathrm{Neg}(e)]\!]^\sharp\, V\, e' \;=\; \begin{cases} \emptyset & \text{if } e' = e \\ V\, e' & \text{otherwise} \end{cases} \\
\ldots &&
\end{array}
$$

$$[\![x = c;]\!]^\sharp\, V\, e' \quad = \quad \begin{cases} (V\, c) \cup \{x\} & \text{if} \quad e' = c \\ (V\, e')\backslash\{x\} & \text{otherwise} \end{cases}$$

$$[\![x = y;]\!]^\sharp\, V\, e \quad = \quad \begin{cases} (V\, e) \cup \{x\} & \text{if} \quad y \in V\, e \\ (V\, e)\backslash\{x\} & \text{otherwise} \end{cases}$$

$$[\![x = e;]\!]^\sharp\, V\, e' \quad = \quad \begin{cases} \{x\} & \text{if} \quad e' = e \\ (V\, e')\backslash\{x\} & \text{otherwise} \end{cases}$$

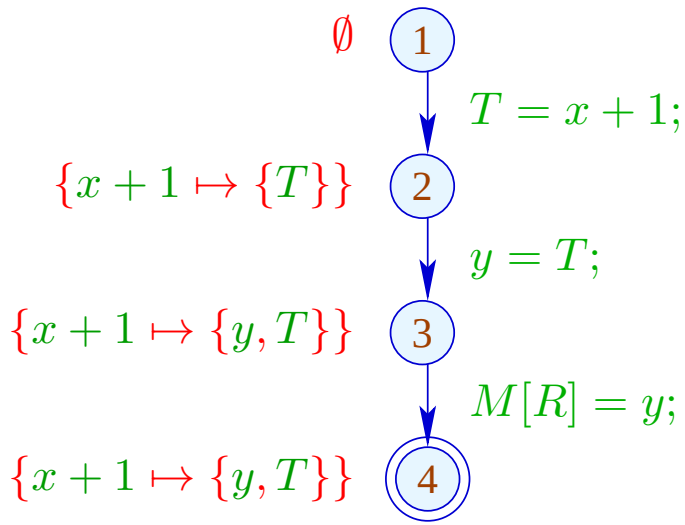
$$[\![x = M[c];]\!]^\sharp\, V\, e' \quad = \quad (V\, e')\backslash\{x\}$$

$$[\![x = M[y];]\!]^\sharp\, V\, e' \quad = \quad (V\, e')\backslash\{x\}$$

$$[\![x = M[e];]\!]^\sharp\, V\, e' \quad = \quad \begin{cases} \emptyset & \text{if } e' = e \\ (V\, e')\backslash\{x\} & \text{otherwise} \end{cases}$$

// analogously for the diverse stores

# In the Example:



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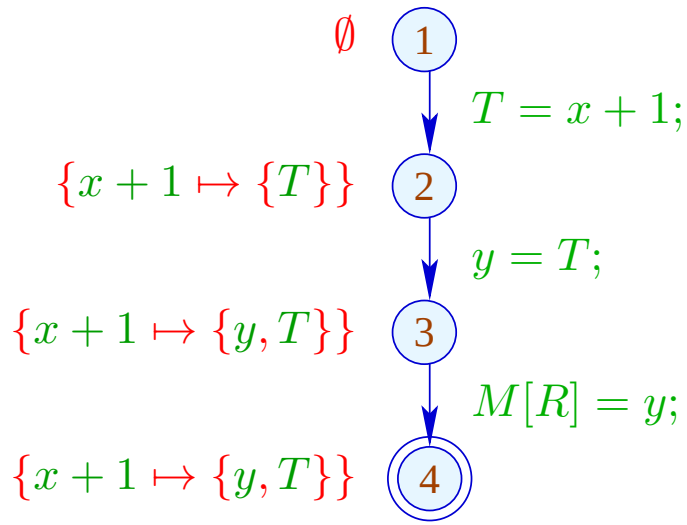



$\rightarrow$ We propagate information in forward direction :-)

At *start*, $V_0\, e = \emptyset$ for all $e$;

$\rightarrow$ $\sqsubseteq \subseteq \mathbb{V} \times \mathbb{V}$ is defined by:

$$V_1 \sqsubseteq V_2 \quad \text{iff} \quad V_1\, e \quad \supseteq \quad V_2\, e \qquad \text{for all} \quad e$$

## Observation:

The new effects of edges are distributive:

To show this, we consider the functions:

(1) $\quad f_1^x\, V\, e = (V\, e) \backslash \{x\}$

(2) $\quad f_2^{e,a}\, V = V \oplus \{e \mapsto a\}$

(3) $\quad f_3^{x,y}\, V\, e = (y \in V\, e)\, ?\, (V\, e \cup \{x\})\, :\, ((V\, e) \backslash \{x\})$

Obviously, we have:

$$\begin{array}{lcl} [\![x = e;]\!]^\sharp & = & f_2^{e,\{x\}} \circ f_1^x \\ [\![x = y;]\!]^\sharp & = & f_3^{x,y} \\ [\![x = M[e];]\!]^\sharp & = & f_2^{e,\emptyset} \circ f_1^x \end{array}$$

By closure under composition, the assertion follows :-))

(1) For $f\ V\ e = (V\ e)\backslash\{x\}$, we have:

$$\begin{aligned}
f\ (V_1 \sqcup V_2)\ e &= ((V_1 \sqcup V_2)\ e)\backslash\{x\} \\
&= ((V_1\ e) \cap (V_2\ e))\backslash\{x\} \\
&= ((V_1\ e)\backslash\{x\}) \cap ((V_2\ e)\backslash\{x\}) \\
&= (f\ V_1\ e) \cap (f\ V_2\ e) \\
&= (f\ V_1 \sqcup f\ V_2)\ e \qquad \text{:-)}
\end{aligned}$$

(2) For $f\,V = V \oplus \{e \mapsto a\}$, we have:

$$
\begin{array}{rcll}
f\,(V_1 \sqcup V_2)\,e' & = & ((V_1 \sqcup V_2) \oplus \{e \mapsto a\})\,e' & \\
& = & (V_1 \sqcup V_2)\,e' & \\
& = & (f\,V_1 \sqcup f\,V_2)\,e' & \text{given that} \quad e \neq e' \\
\\
f\,(V_1 \sqcup V_2)\,e & = & ((V_1 \sqcup V_2) \oplus \{e \mapsto a\})\,e & \\
& = & a & \\
& = & ((V_1 \oplus \{e \mapsto a\})\,e) \cap ((V_2 \oplus \{e \mapsto a\})\,e) & \\
& = & (f\,V_1 \sqcup f\,V_2)\,e & \text{:-)}
\end{array}
$$

(3) For $f\ V\ e = (y \in V\ e)\ ?\ (V\ e \cup \{x\})\ :\ ((V\ e) \backslash \{x\})$, we have:

$$
\begin{array}{rcl}
f\ (V_1 \sqcup V_2)\ e & = & (((V_1 \sqcup V_2)\ e) \backslash \{x\}) \cup (y \in (V_1 \sqcup V_2)\ e)\ ?\ \{x\} : \emptyset \\
& = & ((V_1\ e \cap V_2\ e) \backslash \{x\}) \cup (y \in (V_1\ e \cap V_2\ e))\ ?\ \{x\} : \emptyset \\
& = & ((V_1\ e \cap V_2\ e) \backslash \{x\}) \cup \\
& & ((y \in V_1\ e)\ ?\ \{x\} : \emptyset) \cap ((y \in V_2\ e)\ ?\ \{x\} : \emptyset) \\
& = & (((V_1\ e) \backslash \{x\}) \cup (y \in V_1\ e)\ ?\ \{x\} : \emptyset) \cap \\
& & (((V_2\ e) \backslash \{x\}) \cup (y \in V_2\ e)\ ?\ \{x\} : \emptyset) \\
& = & (f\ V_1 \sqcup f\ V_2)\ e \qquad\qquad \text{:-)}
\end{array}
$$

## We conclude:

- $\rightarrow$ Solving the constraint system returns the MOP solution :-)
- $\rightarrow$ Let $\mathcal{V}$ denote this solution.

  If $x \in \mathcal{V}[u]\, e$, then $x$ at $u$ contains the value of $e$ — which we have stored in $T_e$

  $\Longrightarrow$

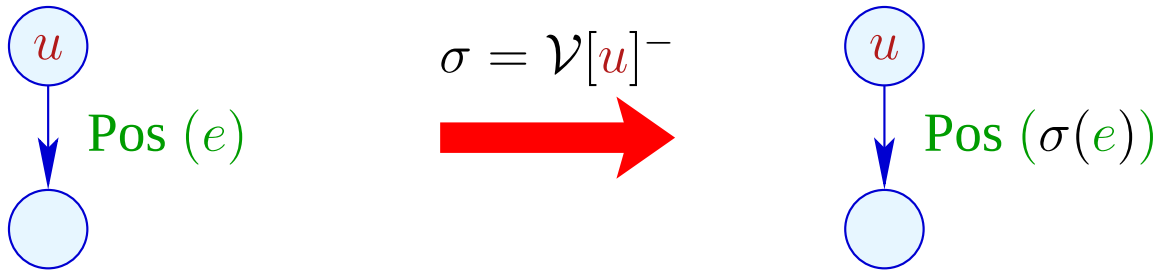
  the access to $x$ can be replaced by the access to $T_e$ :-)

For $V \in \mathbb{V}$, let $V^-$ denote the variable substitution with:

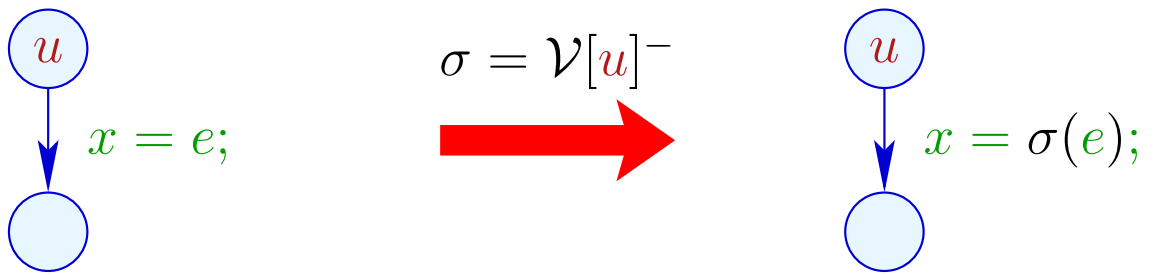
$$V^- x = \begin{cases} T_e & \text{if } x \in V\, e \\ x & \text{otherwise} \end{cases}$$

if $V\, e \cap V\, e' = \emptyset$ for $e \neq e'$. Otherwise: $V^- x = x$ :-)

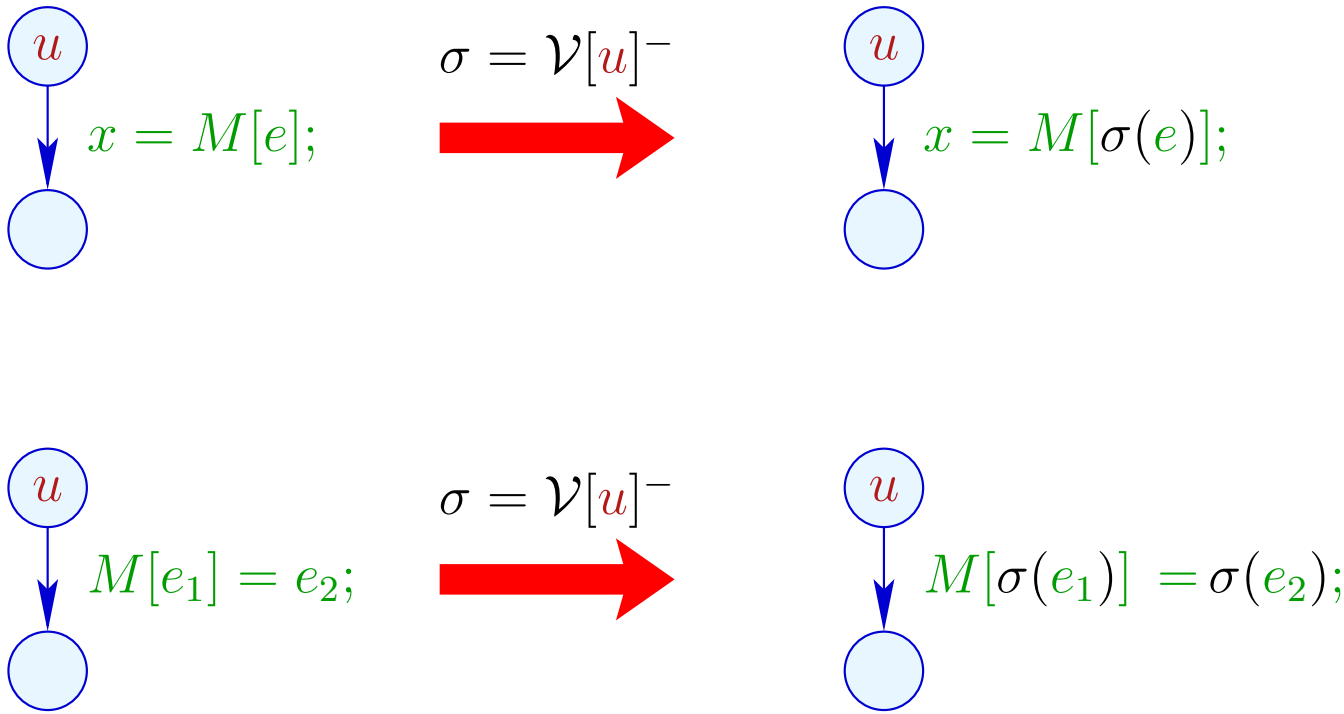
# Transformation 3:




... analogously for edges with Neg $(e)$



# Transformation 3 (cont.):



## Procedure as a whole:

(1) Availability of expressions: T1

- \+ removes arithmetic operations
- – inserts superfluous moves

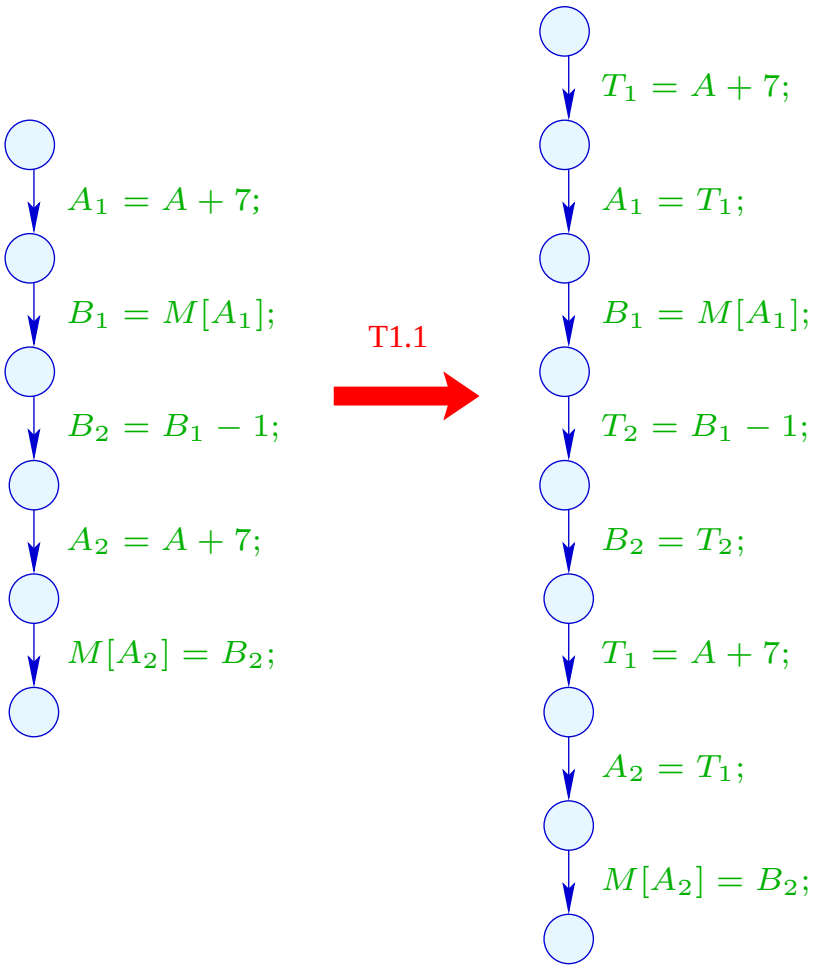
(2) Values of variables: T3

- \+ creates dead variables

(3) (true) liveness of variables: T2

- \+ removes assignments to dead variables

# Example: `a[7]--;`


$A_1 = A + 7;$
$B_1 = M[A_1];$
$B_2 = B_1 - 1;$
$A_2 = A + 7;$
$M[A_2] = B_2;$
T1.1
$T_1 = A + 7;$
$A_1 = T_1;$
$B_1 = M[A_1];$
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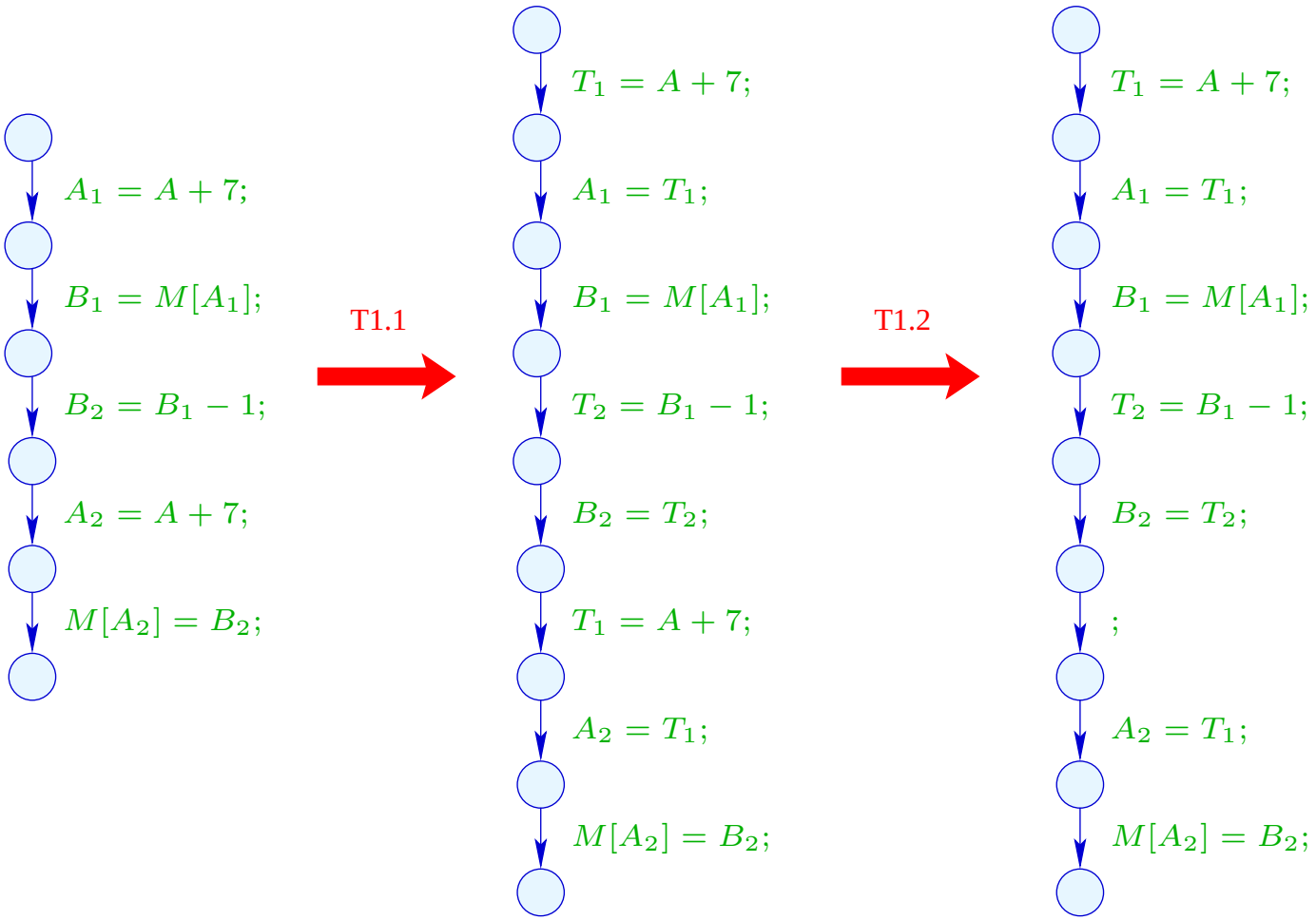
$A_1 = A + 7;$
$B_1 = M[A_1];$
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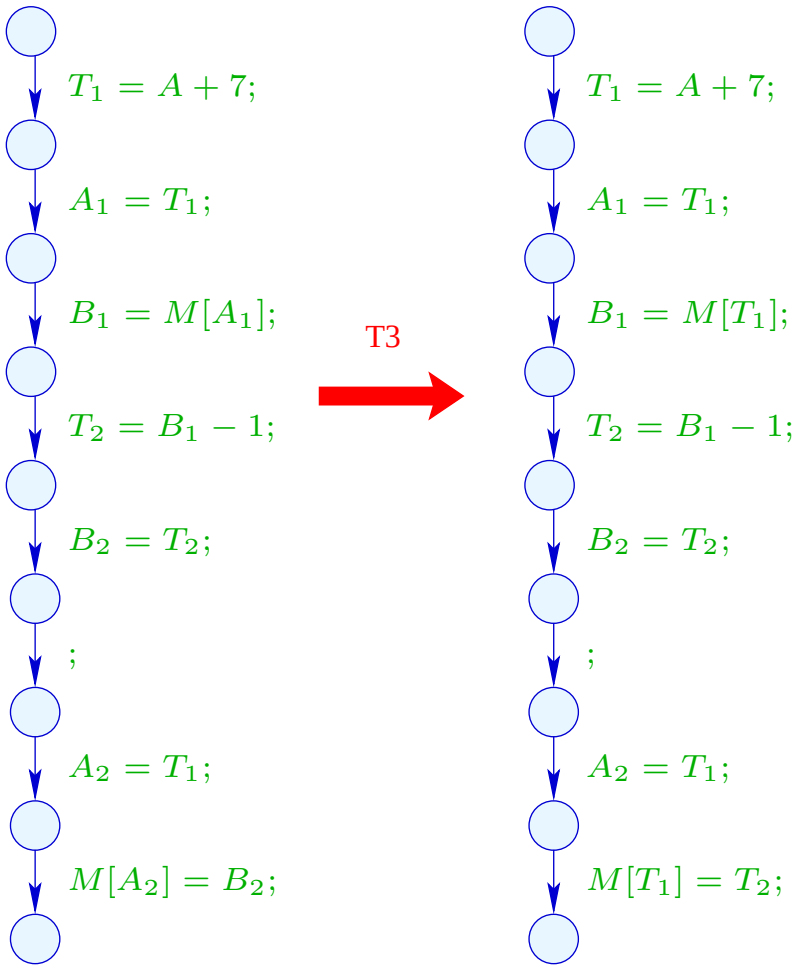
$B_2 = T_2;$
$T_1 = A + 7;$
$A_2 = T_1;$
$M[A_2] = B_2;$
T1.2
$T_1 = A + 7;$
$A_1 = T_1;$
$B_1 = M[A_1];$
$T_2 = B_1 - 1;$
$B_2 = T_2;$
;
$A_2 = T_1;$
$M[A_2] = B_2;$

# Example (cont.): `a[7]--;`



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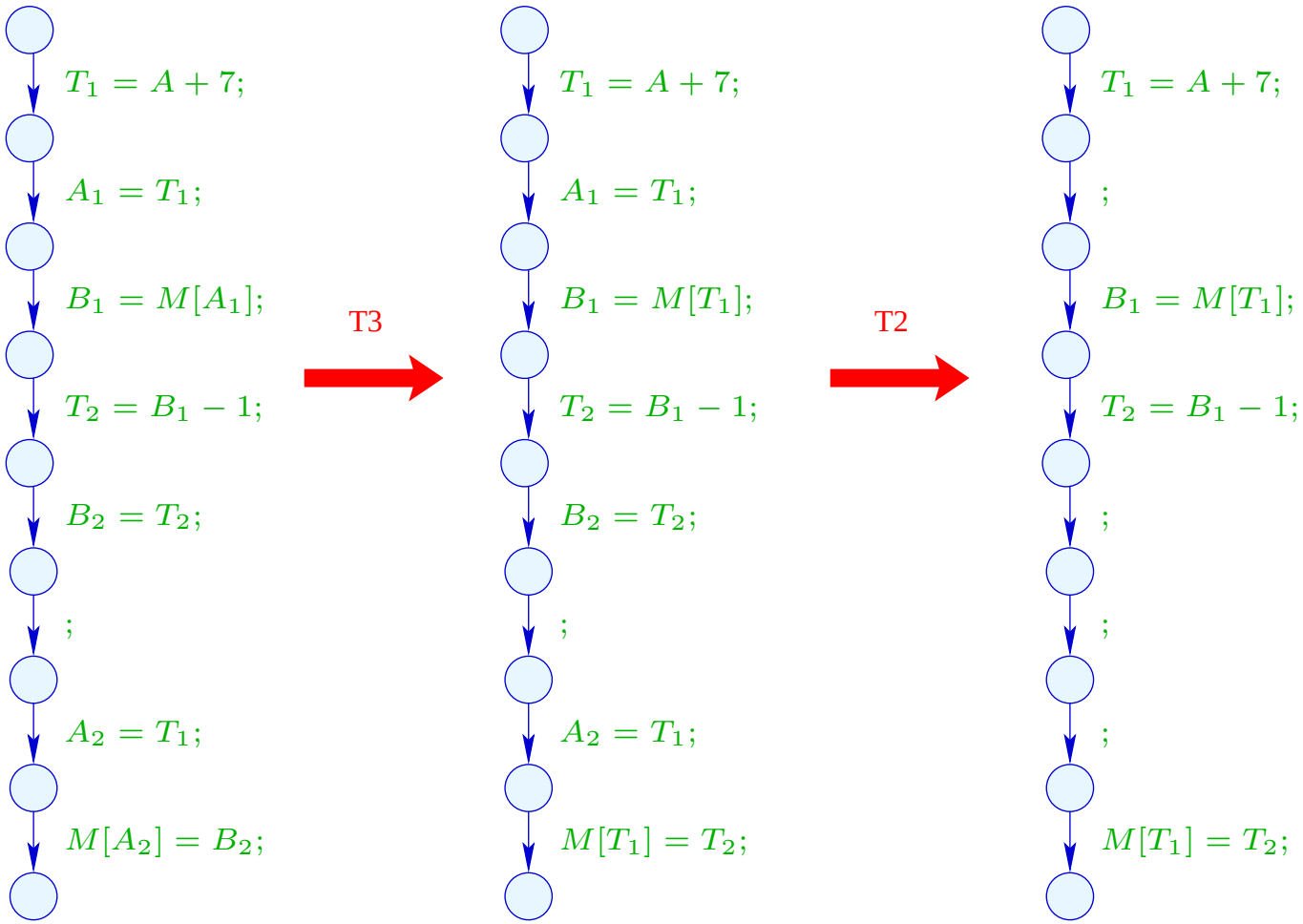


## 1.4 Constant Propagation

### Idea:

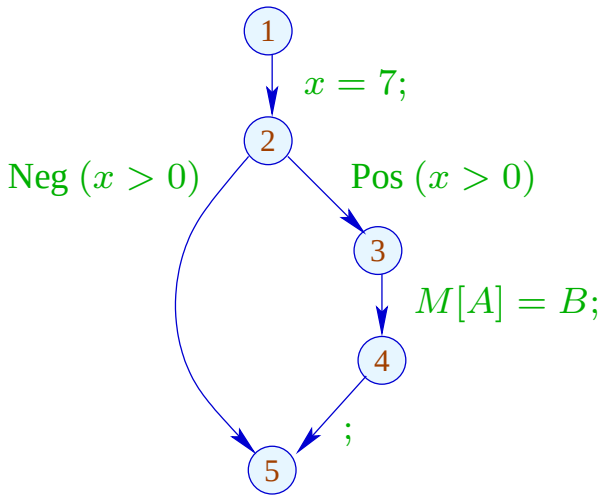
Execute as much of the code at compile-time as possible!

### Example:

$x = 7;$

if $(x > 0)$

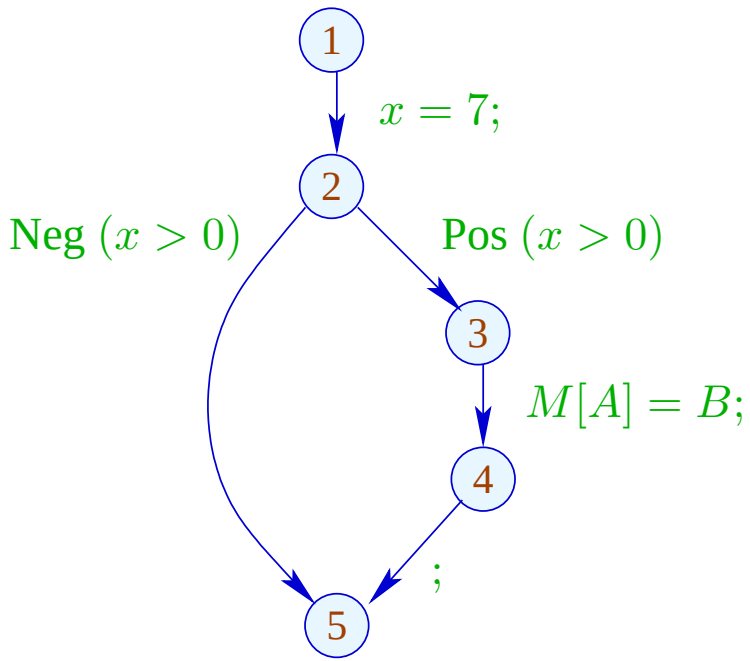
$\quad M[A] = B;$



Obviously, $x$ has always the value 7 :-)

Thus, the memory access is always executed :-))

## Goal:



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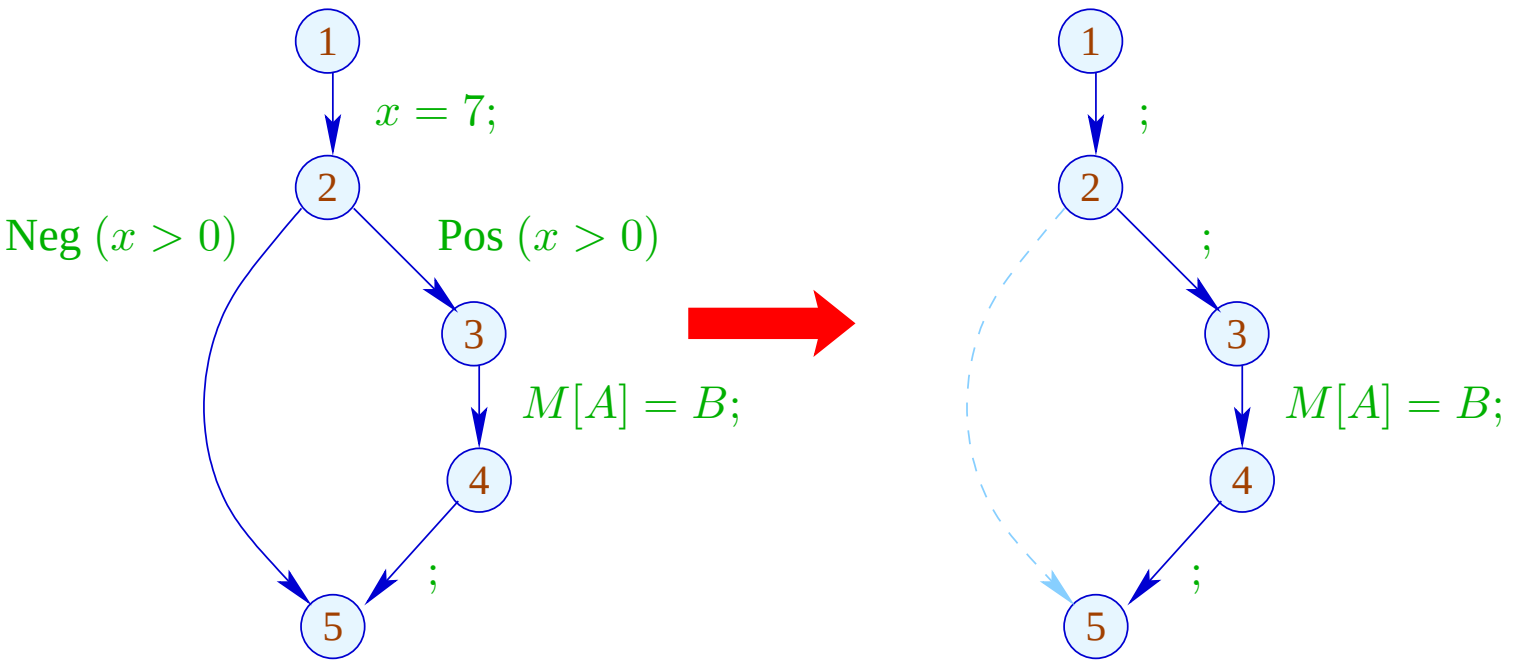

## Generalization: Partial Evaluation

Neil D. Jones, DIKU, Kopenhagen

## Idea:

Design an analysis which for every $u$,

- determines the values which variables definitely have;
- tells whether $u$ can be reached at all :-)

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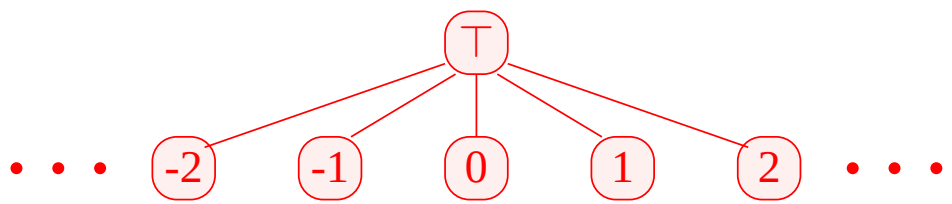
The complete lattice is constructed in two steps.

(1) The potential values of variables:

$$\mathbb{Z}^\top = \mathbb{Z} \cup \{\top\} \qquad \text{with} \quad x \sqsubseteq y \quad \text{iff } y = \top \text{ or } x = y$$



Caveat: $\mathbb{Z}^\top$ is not a complete lattice in itself :-(

(2) $\mathbb{D} = (\mathit{Vars} \to \mathbb{Z}^\top)_\perp = (\mathit{Vars} \to \mathbb{Z}^\top) \cup \{\perp\}$

// $\perp$ denotes: "not reachable" :-))

with $D_1 \sqsubseteq D_2$ iff $\perp = D_1$ or

$D_1\, x \sqsubseteq D_2\, x \quad (x \in \mathit{Vars})$

Remark: $\mathbb{D}$ is a complete lattice :-)

Caveat: $\mathbb{Z}^\top$ is not a complete lattice in itself :-(

(2) $\mathbb{D} = (\mathit{Vars} \to \mathbb{Z}^\top)_\bot = (\mathit{Vars} \to \mathbb{Z}^\top) \cup \{\bot\}$

// $\bot$ denotes: "not reachable" :-))

with $D_1 \sqsubseteq D_2$ iff $\bot = D_1$ or

$D_1\, x \sqsubseteq D_2\, x \quad (x \in \mathit{Vars})$

Remark: $\mathbb{D}$ is a complete lattice :-)

Consider $X \subseteq \mathbb{D}$ . W.l.o.g., $\bot \notin X$ .

Then $X \subseteq \mathit{Vars} \to \mathbb{Z}^\top$ .

If $X = \emptyset$ , then $\bigsqcup X = \bot \in \mathbb{D}$ :-)

If $X \neq \emptyset$ , then $\bigsqcup X = D$ with

$$
\begin{aligned}
D\,x &= \bigsqcup\{f\,x \mid f \in X\} \\
&= \begin{cases} z & \text{if} \quad f\,x = z \quad (f \in X) \\ \top & \text{otherwise} \end{cases}
\end{aligned}
$$

:-))

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\end{aligned}
$$

:-))

For every edge $k = (\_, lab, \_)$ , construct an effect function $[\![k]\!]^\sharp = [\![lab]\!]^\sharp : \mathbb{D} \to \mathbb{D}$ which simulates the concrete computation.

Obviously, $[\![lab]\!]^\sharp \bot = \bot$ for all $lab$ :-)

Now let $\bot \neq D \in Vars \to \mathbb{Z}^\top$.

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$\Longrightarrow$

We must replace the concrete operators $\Box$ by abstract operators $\Box^\sharp$ which can handle $\top$ :

$$a \,\Box^\sharp\, b = \begin{cases} \top & \text{if} \quad a = \top \text{ or } b = \top \\ a \,\Box\, b & \text{otherwise} \end{cases}$$

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- The abstract operators allow to define an abstract evaluation of expressions:

$$[\![e]\!]^\sharp \;:\; (\mathit{Vars} \to \mathbb{Z}^\top) \to \mathbb{Z}^\top$$

Abstract evaluation of expressions is like the concrete evaluation — but with abstract values and operators. Here:

$$\begin{aligned} [\![c]\!]^\sharp\, D &= c \\ [\![e_1 \,\square\, e_2]\!]^\sharp\, D &= [\![e_1]\!]^\sharp\, D \,\square^\sharp\, [\![e_2]\!]^\sharp\, D \end{aligned}$$

... analogously for unary operators :-)

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$$\begin{aligned} [\![c]\!]^\sharp\, D &= c \\ [\![e_1 \,\square\, e_2]\!]^\sharp\, D &= [\![e_1]\!]^\sharp\, D \,\square^\sharp\, [\![e_2]\!]^\sharp\, D \end{aligned}$$

... analogously for unary operators :-)

Example: $D = \{x \mapsto 2, y \mapsto \top\}$

$$\begin{aligned} [\![x + 7]\!]^\sharp\, D &= [\![x]\!]^\sharp\, D \;+^\sharp\; [\![7]\!]^\sharp\, D \\ &= 2 \;+^\sharp\; 7 \\ &= 9 \end{aligned}$$

$$\begin{aligned} [\![x - y]\!]^\sharp\, D &= 2 \;-^\sharp\; \top \\ &= \top \end{aligned}$$

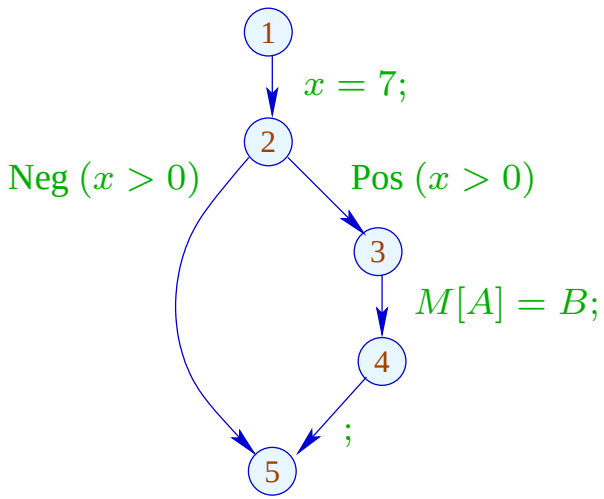
Thus, we obtain the following effects of edges $[\![lab]\!]^\sharp$ :

$$
\begin{array}{lcl}
[\![;]\!]^\sharp\ D & = & D \\
[\![\mathrm{Pos}\,(e)]\!]^\sharp\ D & = & \begin{cases} \bot & \text{if} \quad 0 = [\![e]\!]^\sharp\ D \\ D & \text{otherwise} \end{cases} \\
[\![\mathrm{Neg}\,(e)]\!]^\sharp\ D & = & \begin{cases} D & \text{if} \quad 0 \sqsubseteq [\![e]\!]^\sharp\ D \\ \bot & \text{otherwise} \end{cases} \\
[\![x = e;]\!]^\sharp\ D & = & D \oplus \{x \mapsto [\![e]\!]^\sharp\ D\} \\
[\![x = M[e];]\!]^\sharp\ D & = & D \oplus \{x \mapsto \top\} \\
[\![M[e_1] = e_2;]\!]^\sharp\ D & = & D
\end{array}
$$

... whenever $D \neq \bot$ :-)

At *start*, we have $D_\top = \{x \mapsto \top \mid x \in Vars\}$ .

## Example:



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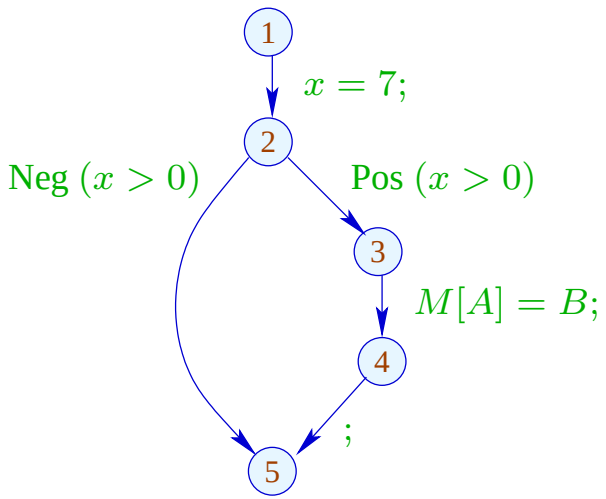


| | |
|---|---|
| 1 | $\{x \mapsto \top\}$ |
| 2 | $\{x \mapsto 7\}$ |
| 3 | $\{x \mapsto 7\}$ |
| 4 | $\{x \mapsto 7\}$ |
| 5 | $\bot \sqcup \{x \mapsto 7\} = \{x \mapsto 7\}$ |

The abstract effects of edges $[\![k]\!]^\sharp$ are again composed to the effects of paths $\pi = k_1 \ldots k_r$ by:

$$[\![\pi]\!]^\sharp = [\![k_r]\!]^\sharp \circ \ldots \circ [\![k_1]\!]^\sharp \quad : \mathbb{D} \to \mathbb{D}$$

Idea for Correctness: Abstract Interpretation

Cousot, Cousot 1977

Patrick Cousot, ENS, Paris