(2) Values of Variables:

- Extend the set *Expr* of expressions by occurring loads $M[e]$.

- Extend the Effects of Edges:

$$[\![x = M[e];]\!]^\sharp \, V \, e' = \begin{cases} \{x\} & \text{if} \quad e' = M[e] \\ \emptyset & \text{if} \quad e' = e \\ V \, e' \backslash \{x\} & \text{otherwise} \end{cases}$$

$$[\![M[e_1] = e_2;]\!]^\sharp \, V \, e' = \begin{cases} \emptyset & \text{if} \quad e' \in \{e_1, e_2\} \\ V \, e' & \text{otherwise} \end{cases}$$

(3) Constant Propagation:

- Extend the abstract state by an abstract store M

- Execute accesses to known memory locations!

$$[\![x = M[e];]\!]^\sharp\,(D, M) \quad = \quad \begin{cases} (D \oplus \{x \mapsto M\,a\}, M) & \text{if} \\ & [\![e]\!]^\sharp\,D = a \sqsubset \top \\ (D \oplus \{x \mapsto \top\}, M) & \text{otherwise} \end{cases}$$

$$[\![M[e_1] = e_2;]\!]^\sharp\,(D, M) \quad = \quad \begin{cases} (D, M \oplus \{a \mapsto [\![e_2]\!]^\sharp D\}) & \text{if} \\ & [\![e_1]\!]^\sharp\,D = a \sqsubset \top \\ (D, \underline{\top}) & \text{otherwise} \end{cases} \qquad \text{where}$$

$$\underline{\top}\,a \quad = \quad \top \qquad\qquad (a \in \mathbb{N})$$

Problems:

- Addresses are from $\mathbb{N}$:-(

  There are no infinite strictly ascending chains, but ...
- Exact addresses at compile-time are rarely known :-(
- At the same program point, typically different addresses are accessed ...
- Storing at an unknown address destroys all information M :-(

$\Longrightarrow$ constant propagation fails :-(

$\Longrightarrow$ memory accesses/pointers kill precision :-(

Simplification:

- We consider pointers to the beginning of blocks A which allow indexed accesses $A[i]$:-)
- We ignore well-typedness of the blocks.
- New statements:

$$x = \mathsf{new}(); \quad // \quad \text{allocation of a new block}$$

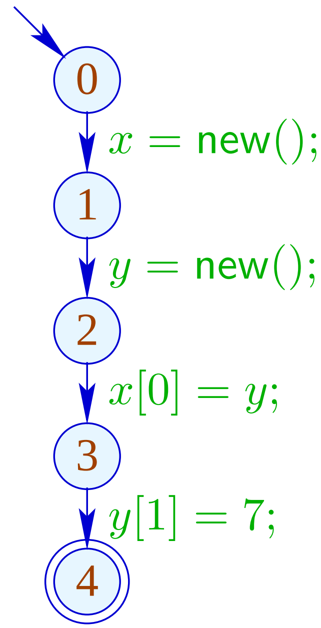
$$x = y[e]; \quad // \quad \text{indexed read access to a block}$$

$$y[e_1] = e_2; \quad // \quad \text{indexed write access to a block}$$

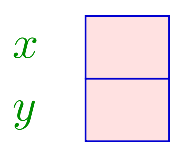
- Blocks are possibly infinite :-)
- For simplicity, all pointers point to the beginning of a block.

Simple Example:

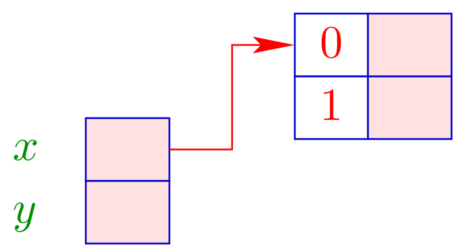
$$x = \mathsf{new}();$$
$$y = \mathsf{new}();$$
$$x[0] = y;$$
$$y[1] = 7;$$



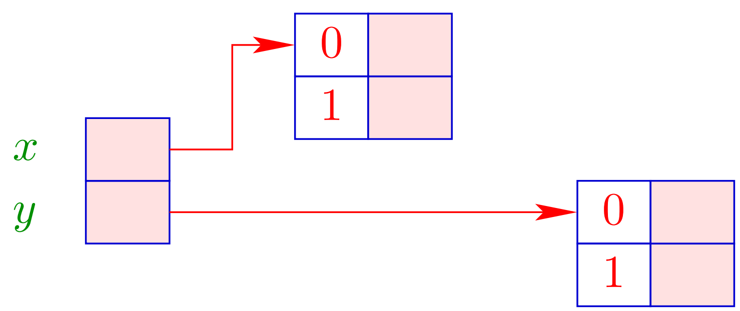
The Semantics:



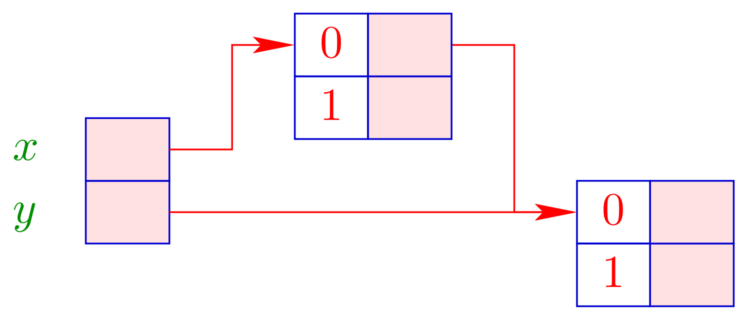
The Semantics:



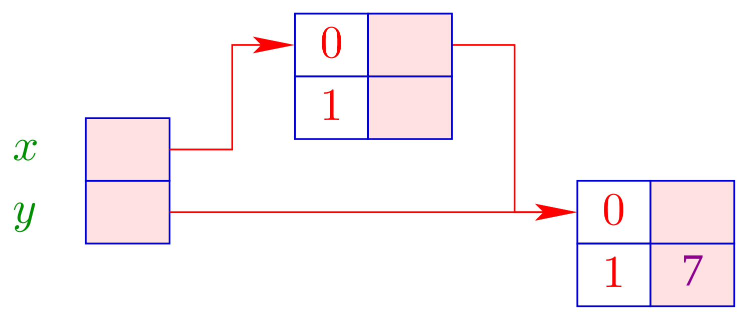
The Semantics:



The Semantics:



The Semantics:



More Complex Example:

$r = \textsf{Null};$

$\textsf{while}\ (t \neq \textsf{Null})\ \{$

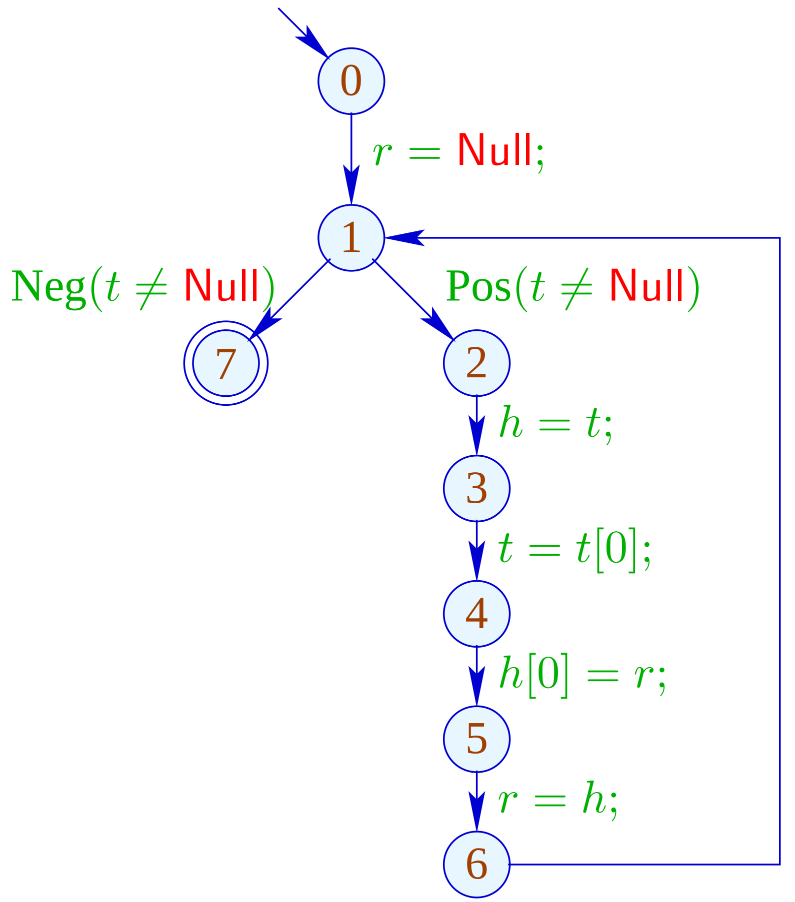
$\quad h = t;$

$\quad t = t[0];$

$\quad h[0] = r;$

$\quad r = h;$

$\}$



Concrete Semantics:

A store consists of a finite collection of blocks.

After h new-operations we obtain:

$$\begin{array}{llll} \mathit{Addr}_h & = & \{\mathsf{ref}\ a \mid 0 \leq a < h\} & // \text{ addresses} \\ \mathit{Val}_h & = & \mathit{Addr}_h \cup \mathbb{Z} & // \text{ values} \\ \mathit{Store}_h & = & (\mathit{Addr}_h \times \mathbb{N}_0) \to \mathit{Val}_h & // \text{ store} \\ \mathit{State}_h & = & (\mathit{Vars} \to \mathit{Val}_h) \times \mathit{Store}_h & // \text{ states} \end{array}$$

For simplicity, we set: $0 = \mathsf{Null}$

Let $\quad (\rho, \mu) \in State_h$. Then we obtain for the new edges:

$$
\begin{array}{lll}
[\![x = \mathsf{new}();]\!]\,(\rho, \mu) & = & (\rho \oplus \{x \mapsto \mathsf{ref}\; h\}, \\
& & \quad \mu \oplus \{(\mathsf{ref}\; h, i) \mapsto 0 \mid i \in \mathbb{N}_0\}) \\
[\![x = y[e];]\!]\,(\rho, \mu) & = & (\rho \oplus \{x \mapsto \mu\,(\rho\, y, [\![e]\!]\,\rho)\}, \mu) \\
[\![y[e_1] = e_2;]\!]\,(\rho, \mu) & = & (\rho, \mu \oplus \{(\rho\, y, [\![e_1]\!]\,\rho) \mapsto [\![e_2]\!]\,\rho\})
\end{array}
$$

Caveat:

This semantics is too detailled in that it computes with absolute Addresses. Accordingly, the two programs:

$$x = \mathsf{new}(); \qquad\qquad y = \mathsf{new}();$$
$$y = \mathsf{new}(); \qquad\qquad x = \mathsf{new}();$$

are not considered as equivalent !!?

Possible Solution:

Define equivalence only up to permutation of addresses :-)

Alias Analysis 1. Idea:

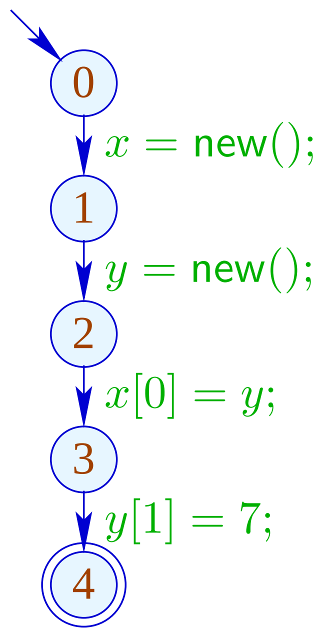
- Distinguish finitely many classes of blocks.
- Collect all addresses of a block into one set!
- Use sets of addresses as abstract values!

$\Longrightarrow$ Points-to-Analysis

$$
\begin{array}{lcll}
Addr^\sharp & = & Edges & \text{// creation edges} \\
Val^\sharp & = & 2^{Addr^\sharp} & \text{// abstract values} \\
Store^\sharp & = & Addr^\sharp \to Val^\sharp & \text{// abstract store} \\
State^\sharp & = & (Vars \to Val^\sharp) \times Store^\sharp & \text{// abstract states}
\end{array}
$$

// complete lattice !!!

... in the Simple Example:



	x	y	$(0,1)$
0	$\emptyset$	$\emptyset$	$\emptyset$
1	$\{(0,1)\}$	$\emptyset$	$\emptyset$
2	$\{(0,1)\}$	$\{(1,2)\}$	$\emptyset$
3	$\{(0,1)\}$	$\{(1,2)\}$	$\{(1,2)\}$
4	$\{(0,1)\}$	$\{(1,2)\}$	$\{(1,2)\}$

The Effects of Edges:

$$
\begin{array}{lcl}
[\![(\_, ;, \_)]\!]^\sharp \, (D, M) & = & (D, M) \\
[\![(\_, \mathrm{Pos}(e), \_)]\!]^\sharp \, (D, M) & = & (D, M) \\
[\![(\_, x = y;, \_)]\!]^\sharp \, (D, M) & = & (D \oplus \{x \mapsto D\, y\}, M) \\
[\![(\_, x = e;, \_)]\!]^\sharp \, (D, M) & = & (D \oplus \{x \mapsto \emptyset\}, M) \qquad , \qquad e \notin \mathit{Vars} \\
\\
[\![(u, x = \mathsf{new}();, v)]\!]^\sharp \, (D, M) & = & (D \oplus \{x \mapsto \{(u, v)\}\}, M) \\
[\![(\_, x = y[e];, \_)]\!]^\sharp \, (D, M) & = & (D \oplus \{x \mapsto \bigcup\{M(f) \mid f \in D\, y\}\}, M) \\
[\![(\_, y[e_1] = x;, \_)]\!]^\sharp \, (D, M) & = & (D, M \oplus \{f \mapsto (M\, f \cup D\, x) \mid f \in D\, y\})
\end{array}
$$

Caveat:

- The value Null has been ignored. Dereferencing of Null or negative indices are not detected :-(
- Destructive updates are only possible for variables, not for blocks in storage!

  $\Longrightarrow$ no information, if not all block entries are initialized before use :-((
- The effects now depend on the edge itself.

  The analysis cannot be proven correct w.r.t. the reference semantics :-(

  In order to prove correctness, we first instrument the concrete semantics with extra information which records where a block has been created.

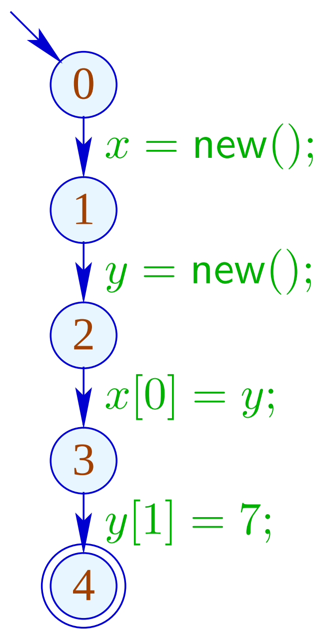
...

- We compute possible points-to information.
- From that, we can extract may-alias information.
- The analysis can be rather expensive — without finding very much :-(
- Separate information for each program point can perhaps be abandoned ??

2. Idea:

Compute for each variable and address a value which safely approximates the values at every program point simultaneously !

... in the Simple Example:



x	$\{(0,1)\}$
y	$\{(1,2)\}$
$(0,1)$	$\{(1,2)\}$
$(1,2)$	$\emptyset$

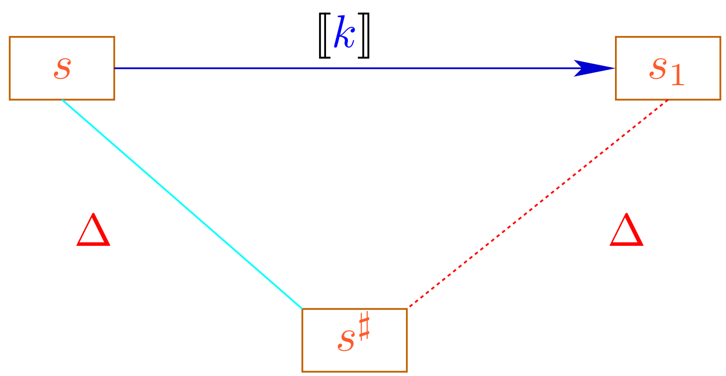
Each edge (u, lab, v) gives rise to constraints:

lab	$Constraint$
$x = y;$	$\mathcal{P}[x] \supseteq \mathcal{P}[y]$
$x = \mathsf{new}();$	$\mathcal{P}[x] \supseteq \{(u, v)\}$
$x = y[e];$	$\mathcal{P}[x] \supseteq \bigcup\{\mathcal{P}[f] \mid f \in \mathcal{P}[y]\}$
$y[e_1] = x;$	$\mathcal{P}[f] \supseteq (f \in \mathcal{P}[y])\, ?\, \mathcal{P}[x] \ :\ \emptyset$
	for all $f \in Addr^\sharp$

Other edges have no effect :-)

Discussion:

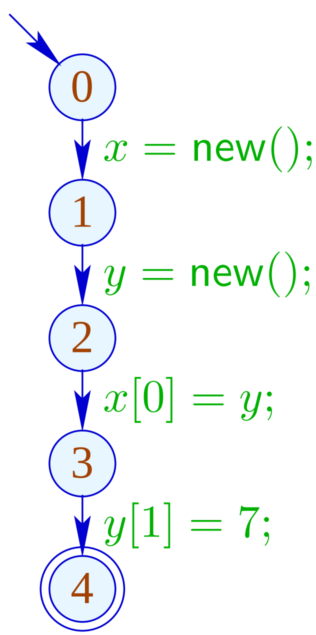
- The resulting constraint system has size $\mathcal{O}(k \cdot n)$ for k abstract addresses and n edges :-(
- The number of necessary iterations is $\mathcal{O}(k \cdot (k + \# Vars))$ ...
- The computed information is perhaps still too zu precise !!?
- In order to prove correctness of a solution $s^\sharp \in States^\sharp$ we show:



Alias Analysis 3. Idea:

Determine one equivalence relation $\equiv$ on variables x and memory accesses $y[\,]$ with $s_1 \equiv s_2$ whenever s_1, s_2 may contain the same address at some u_1, u_2

... in the Simple Example:



$$\equiv \;=\; \{\{x\},\; \{y, x[\,]\},\; \{y[\,]\}\}$$

Discussion:

$\rightarrow$ We compute a single information fo the whole program.

$\rightarrow$ The computation of this information maintains partitions $\pi = \{P_1, \ldots, P_m\}$:-)

$\rightarrow$ Individual sets P_i are identified by means of representatives $p_i \in P_i$.

$\rightarrow$ The operations on a partition π are:

$$
\begin{array}{lll}
\mathsf{find}\,(\pi, p) & = & p_i \qquad\qquad \text{if } p \in P_i \\
 & // & \text{returns the representative} \\
\mathsf{union}\,(\pi, p_{i_1}, p_{i_2}) & = & \{P_{i_1} \cup P_{i_2}\} \cup \{P_j \mid i_1 \neq j \neq i_2\} \\
 & // & \text{unions the represented classes}
\end{array}
$$

→ If $x_1, x_2 \in \mathit{Vars}$ are equivalent, then also $x_1[\,]$ and $x_2[\,]$ must be equivalent :-)

→ If $P_i \cap \mathit{Vars} \neq \emptyset$, then we choose $p_i \in \mathit{Vars}$. Then we can apply union recursively :

$$
\begin{array}{lll}
\mathsf{union}^*\,(\pi, q_1, q_2) & = & \mathsf{let}\;\; p_{i_1} \;=\; \mathsf{find}\,(\pi, q_1) \\
& & \qquad\; p_{i_2} \;=\; \mathsf{find}\,(\pi, q_2) \\
& & \mathsf{in}\;\; \mathsf{if}\; p_{i_1} == p_{i_2}\; \mathsf{then}\; \pi \\
& & \qquad \mathsf{else}\;\; \mathsf{let}\; \pi \;=\; \mathsf{union}\,(\pi, p_{i_1}, p_{i_2}) \\
& & \qquad\qquad \mathsf{in}\;\; \mathsf{if}\; p_{i_1}, p_{i_2} \in \mathit{Vars}\; \mathsf{then} \\
& & \qquad\qquad\quad \mathsf{union}^*\,(\pi, p_{i_1}[\,], p_{i_2}[\,])
\end{array}
$$